ESTIMATION OF TREE VOLUME EQUATIONS

by

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STATEMENT

Except where otherwise acknowledged, to the best of my knowledge, the work described in this substantial essay is original and has not been submitted for any degree or diploma in any University.

(P.N. RAY)

ABSTRACT

Accurate estimation of tree volume is the primary objective of forest inventory. Amongst various methods of estimation, statistical methods using prediction models offer the greatest gain in terms of precision. In this study certain aspects of tree volume models have been explored.

Tree volume data regarding gross merchantable volume under bark of 5 eucalypt species were collected from Victoria. This data included both over bark and under bark measurements. Tree volume functions were fitted separately to diameter over bark and total height (tree height), diameter under bark and tree height and diameter over bark and merchantable height measurements. The regression package GLIM (release-3) was used for fitting and testing the volume models. Error variance was found to be heterogenous and thus weighting had to be performed. Exponential error variance functions were developed initially for each Species Group but further analysis indicated that a common function was applicable to all species. Weighted regression models were then developed using relevant weighting function.

Grouping of species for volume estimation was studied by using covariance analysis with dummy variables. Smooth-barked species could be successfully grouped and so could the rough-barked ones. The final volume model selected in the case of measurements with diameter over bark and tree height took the following form:

$$V = b_o + b_1H + b_2D^2 + b_3 H^2 + b_4 D^2H$$

In the case of models using diameter under bark and tree height the following form was selected:

$$V = b_o + b_1 D^2H + b_2 D^3H + b_3 (D^2H)^2$$

In the case of models with diameter over bark and merchantable height the form selected was:

$$V = b_o + b_1 H + b_2 D^2H + b_3 H^2 + b_4 D^2H^2$$

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CHAPTER 1

INTRODUCTION

1.1 Importance of Accurate Information

Accurate inventories are important for forest management decisions. Nilsson (1973) defines forest inventory as "How much industrial wood, by species, dimensions and grades can be made available at certain periods and at tentative mill-sites within alternative cost limits per volume unit?" A significant part of forest management costs in recent years has been devoted to the inventory of standing timber and pattern of stand structure (Bamping, 1974). Recent developments in biometric techniques and computing facilities have opened up new vistas in forest management. Management options have also become more varied and are judged mainly in financial terms.

1.2 Importance of Tree Volume Information for Tropical Forests

Tropical forests cover a total area of approximately 19 million km^2 or 14 percent of the world's area and slightly less than half of the world's forested area (Howard and Lanly, 1975). In view of the rising demand for wood and wood products and the depletion of traditional timber resources of temperate countries, the resources of tropical forests in developing countries are assuming greater importance.

Techniques of forest inventory in the tropics are less developed than for temperate forests.

Table 1.1 ESTIMATED DEMAND FOR INDUSTRIAL WOOD IN INDIA

(in million m^3 R.W.E.)

	1980	1985	2000
Low income growth	25	30	47
High income growth	27	35	64

Source: Government of India, 1976.

Precise data regarding composition of forests, quantity of industrial wood and growth rates are not known (King, 1976). Aspects of tree and stand volume estimation in tropical forests have been studied less due to great variability in the exploitable volume of trees and other related difficulties. One of the consequences of rapid economic development within these countries is that wood resources are being more intensively utilised, and hitherto inoperable areas now being exploited. Accurate estimation of potentially available wood resources from tropical forests has become crucial in the management of such forests.

In India, the industrial wood requirement for 1985 and 2000 has been estimated both for low income growth and high income growth, as shown in Table 1.1. The ability of existing potentially exploitable forests to meet India's future demand can only be ascertained by accurate estimation of growing stock. There has been mounting interest in the last two decades in the development of forest-based industries like pulp, paper and processed wood. Assessment of the volume of utilisable wood expected to be obtained from 75.03 million ha of forests under the control of different states and Union territories has become an important aspect of national forest policy and in planning industrial development. Some 50.8 percent of forest areas have been covered under regular working plans or schemes which survey the growing stock at intervals of 10 years.

Until recently, general volume tables and local volume tables constituted the only available means of determining the volume of standing trees. Derivation of these tables has mainly been graphical, though in more recent work, statistical techniques through standard regression methods have been adopted. There is considerable scope for improvement in these volume tables, especially with respect to their statistical properties. In India the modern volume equation approach has been adopted by the Preinvestment Survey of Forest Resources, in the investigation of the raw material supply from existing and potential forest resources. In a survey of 39.6 million ha (G.O.I., 1979) by the organisation, voluminous data on felled trees of different commercially important species were collected using various statistical designs. These data can be used in the application of modern techniques of volume estimation, thereby refining and improving existing volume tables.

1.3 Scope and Aims of the Study

In this study various aspects of tree volume estimation will be considered. Amongst various techniques of tree volume estimation, statistical methods offer the greatest gain in terms of providing precise, unbiased predictions. The data used to illustrate these methods consist of 125 trees of 5 different species of regrowth eucalypts from Victorian forests. The 5 species of eucalypt differ considerably in their morphological characteristics; some being rough-barked, others smooth. Data are available for both over bark and under bark and for 'point' height and total height. Grouping of species for the purpose of volume estimation will be considered in this study because of the importance in tropical forests, where "often there will be more species of trees in a few acres than for the entire European flora" (Odum, 1971).

The aims of this study are therefore to explore and illustrate the most efficient means of estimating tree volume equations.

CHAPTER 2

VOLUME PREDICTION

2.1 Methods of Volume Prediction

An estimate of the volume of standing trees in a forest can be obtained from an inventory. This estimate may be based either on complete enumeration or on sampling of the volume of individual trees. Stand volume can also be estimated from stand characteristics using an appropriate stand volume function. Complete enumeration of a forest is time consuming, expensive and is used only in high value forests e.g. tropical peat swamp forests of Malaysia (Brunig, 1963), sandalwood and rosewood forests of India. In most forests complete enumeration is impractical, and it is necessary to derive volume estimates by sampling. Here the intensity of sampling on an area basis may be very low, even very much less than 1 percent. Often, within the selected sample units, further sub-sampling is necessary to reduce the cost of inventory to an acceptable level. Sampling is a reasonable alternative when the consequences of error introduced by it cost less than the added effort of complete enumeration (Bickford, 1966).

After sample units have been selected, their volume may be measured directly or estimated using established volume functions. Methods of estimating tree volume in forests can be grouped broadly into two types, viz 1) Ground methods and 2) methods using remote sensing. The present study is confined to ground methods.

2.2 Tree Volume Tables

2.2.1 Definition

A volume table is essentially a statement of the expected volume of a standing tree of particular dimensions in a particular stand or population (Wood, 1980). It provides an estimate of the average volume for trees of a particular dimension in diameter, height and similar variables. The estimate for an individual tree is likely to be less accurate than that obtained by actual measurement.

The common dependent variables used are, total volume over bark (o.b.) or under bark (u.b.), volume up to a certain diameter limit (merchantable volume), bole volume and pruned section volume.

The method involves measuring a few easily and directly observable dimensions of a tree. The volume of the tree is then estimated from a previously established relationship between these independent variables and volume. The independent variables commonly used are, diameter over bark (d.o.b.) or diameter under bark (d.u.b.), girth at breast height (g.b.h.), height (total or merchantable), bark thickness and some measure of form.

The concept of form factor as an index of tree form is rather theoretical, because direct measurement of form on a standing tree is not possible (Spurr, 1952). The form quotient approach measures the ratio between two diameters measured at different heights on tree bole. Nevertheless, Golding and Hall (1961) tested 25 tree volume prediction models for North American conifers and found that with tree form as a third variable, the precision of estimate increased significantly. Ilvessalo (1947) in Finland, and Cromer, McIntyre and Lewis (1955) in Australia used taper, instead of form factor. Though inclusion of a measure of taper is aimed to improve precision of prediction, in practice such a measure may not properly relate the effect of entire tree taper on volume as it mainly consists of the ratio of diameters of the tree sufficiently close to the ground. Smith et al. (1961) studied various form expressions in certain conifers and concluded "no practical advantage was to be gained from the use of any measure of form in addition to d.b.h. and total height".

Bark thickness may also be considered as an additional variable. To be efficient, it should be reliably measured and the variation in it must be correlated with variation in volume. Cromer et al. (1955) used bark thickness in addition to other variables e.g. diameter at breast height (d.b.h.), height and measurement of taper in a volume function of *Pinus radiata* in plantation. However this is of limited use particularly in hardwoods.

2.2.2 Classification

A single independent variable or set of variables may be used in the table depending on the precision of estimate desired, variability of the population, cost and other factors. Generally as the number of independent variables is increased, the reliability of the volume estimate also increases. Volume tables may be classified as 1-way (single entry), 2-way (double entry) etc, depending on the number of independent variables used. The conventional nomenclature of local, standard, regional, and general or universal table, indicating the scope of application and implied number of independent variables used, is unsound as the scope of application and number of independent variables need not be related.

Single entry tree volume tables usually relate tree volume to d.b.h.o.b., g.b.h.o.b. or basal area over bark (b.a.o.b.). Height is sometimes taken into account during construction but is eliminated in the final relationship. Such 1-way volume tables have been reported to be satisfactory for three species in Rhodesia for which bole length was more or less constant in trees of merchantable size (Banks and Burrows, 1966). In the case of a given species or a group of species of some mixed tropical hardwoods, the average bole height (from top of buttress) to crown point in every diameter class above a certain diameter was almost constant (FAO, 1973) in which case the inclusion of height was not necessary. A composite volume table for total volume (bole plus branch wood) has been found acceptable in the low land Dipterocarp forests of Malaysia (Wong, 1966). A similar table has been constructed for teak (Chaturvedi, 1973) and for the total volume of 10 native tree species of India (Sharma and Jain, 1978; 1979). In general, however, these 1-way tables have local applicability only.

The most commonly used volume tables or functions include two variables - d.b.h. and some expression of height. Such tables are prepared for individual species or a group of species and for specific localities. The literature on these types of volume functions are voluminous and have been reviewed by Spurr (1952). Use of such a table requires measurement of height of every tree within the sampling unit or at least a sample of them. At times, the increased cost of measuring heights is not commensurate with reduction of sampling error. In such cases the use of a one way table may be advisable.

2.2.3 Methods of volume table construction

The normal procedure of volume table construction is to relate volume directly to height and diameter by means of graphs, alignment charts or prediction models.

2.2.3.1 *Graphic techniques*

These, the oldest methods of volume table construction, comprise three major types viz,

a) Volume curve: It is based on establishing relationship between volume and d.b.h.o.b. which is mainly curvilinear with curve concave upwards. This method has been in use in Western Europe for about a hundred and fifty years (Carron, 1968) and in India for a long time.

b) Volume line: This is a simple relationship between volume and basal area. Because the relationship of volume to d.b.h.o.b. tends to be parabolic, linear transformation of this by plotting volume to basal area proved to be useful. It has been investigated and used a great deal in Great Britain and Australia. The shape of the volume curve is a second degree parabola for total volume (u.b.) or merchantable volume (u.b.) to 15cm or less in many plantation conifers in Australia (Wood, 1980). In the case of young stands, specially those on poor site quality and for old unthinned stands and for tree volume (up to small end d.u.b., greater than 15cm) the volume-basal area relationship is curvilinear (Carron, 1968). As the limit of diameter (u.b.) increases, the curvilinearity also increases. A linear relationship of volume (u.b.) and b.a.o.b. also holds for evenaged stands of a number of eucalypt species e.g. regrowth blackbutt at Pine Creek State Forest, New South Wales. Thus, the volume line for a particular stand is the 1-way volume table for the stand. A strong linear relationship between volume and basal area facilitates easy fitting of data to a straight line and fewer observations are needed.

c) <u>Tariff</u>: A tariff comprises a series of volume curves or lines relating d.b.h., tariff number and volume. It furnishes a 1-way table for a given stand. A feature of any pure evenaged stand is that if the volume of each tree is plotted graphically against its basal area, the points lie on a straight line. The slope of the line varies depending on species, age and height, but all these tariff lines converge at a common point and each of these lines can be converted to a single-entry volume table, each tariff table representing one such line. The system proved to be efficient for the Forestry Commission, U.K. and has been in operation since 1956 (Hamilton, 1975). This method has also been used in South Australia and in Queensland.

Spurr (1952) and Carron (1968) have mentioned the advantages of graphical techniques. These methods are simple and require less mathematical background for their implementation. However, disadvantages are substantial. The techniques are highly subjective with consequent danger of personal bias, and a large number of tree data are required to fit curves accurately. Handling more than two independent variables becomes difficult and is generally impractical. No statistically valid method exists for estimating precision of volume estimates derived using these technques. For these reasons, these graphical or semigraphical methods should no longer be used for table construction (FAO, 1973). Such methods are being replaced by models derived from statistical analysis.

2.2.3.2 *Volume tables derived by statistical analysis*

Tree or stand volume tables may also be derived from volume equations expressing volume as a function of diameter, height or other practically measurable parameters. Such equations are derived from statistical regression analysis which will be discussed in some detail later in the study.

Various equations have been investigated with the object of establishing functional relationship between volume, d.b.h. and height. Some commonly used tree volume equations are:-

1) Logarithmic equation (Schumacher and Hall, 1933)

$\log V = \log a + b \log D + c \log H$

The fundamental concept here being that volume of tree can be expressed by the formula $V = aD^bH^c$, where V = Volume, D = D.b.h.,

H = Height, a is a constant and b and c are the exponents to which the independent variables are raised. Thornber (1948) suggested that since volume is a cubic function, the sum of exponents of D and H should be equal to three and proposed a modified equation. Brown (1962) used this to constuct volume tables for trees in British Columbia. An imprecise measure of upper tree diameter (D_u) was incorporated (Spurr, 1952) in the original logarithmic equation and it resulted in the logarithmic form diameter formula, $\log V = \log a + b \log D + c \log H + d \log D_u$. Further various modifications of the logarithmic equation had been developed and these had been evaluated by Spurr (1952). He pointed out that the use of logarithms resulted in consistent under-estimation of actual volume. He also concluded that such models were less precise than arithmetic ones and he did not recommend them.

2) Combined variable equation (Spurr, 1952)

$$V = b_o + b_1 D^2H$$

The combined variable equation is by far the simplest and most widely used. This equation is generally attributed to Spurr but better attributed to Naslund (Leech, 1973). It is based on the premise that total volume of trees are known to be roughly proportional to the product of their total height and d.b.h.-squared. It does not assume a constant form factor. Here relationship between tree form factor and D^2H or product of basal area and height is assumed to be hyperbolic. Spurr obtained excellent results by using this equation in case of North American species. In case of Canadian species similar results were reported by Golding and Hall (1961) who tested 25 models. In case of tropical forest trees also it showed high precision (Hindley, 1973). This model was used for volume estimation of trees of Sarawak forests. Spurr recommended this simple equation for very few data (50-100 trees) with a check for curvilinearity by plotting volume over D^2H on graphs.

3) Australian Equation (Stoate, 1945)

$$V = b_o + b_1 D^2 + b_2 H + b_2 D^2H$$

The use of this equation was first proposed by Stoate (1945). Spurr (1952) tested this equation on limited data and found it to be better than the combined variable equation for some data. Under Australian conditions with plantation conifers this equation is satisfactory in the case of total and merchantable volume generally to 10cm and sometimes to 15cm diameter (u.b.) (Wood, 1980). It has been widely used (e.g. Cromer et al. 1955; Henry 1960).

Other well known equations can be grouped into 1) Arithmetic standard or nonform class equations, 2) Logarithmic form class equations etc and are discussed by Spurr (1952) and Husch (1963).

With the widespread use of digital computers and development of biometric techniques, mensurationists have begun to explore more complicated equations.

2.2.4 Methods using remote sensing

Remote sensing is the acquisition of information about objects through a remote device not being in contact with the objects. Though of comparatively recent origin, this technique is developing rapidly and there have been spectacular achievements in the field of forest resource inventory. Besides conventional aerial photography the technique now embraces various active and passive sensors in the visible, infra-red and microwave regions of the electromagnetic spectrum. Methods developed for estimating volume of trees need mention.

2.2.4.1 *Aerial tree volume table*

Aerial tree volume tables are used to determine the volume of individual trees from air-photos. These tables are based on the relation between d.b.h. and crown diameter. Various workers as early as 1928 tried this approach (Zeiger, 1928, cited by Spurr, 1952). An approximate estimate of volume can be obtained from existing d.b.h.-volume relationship after regression of d.b.h. on crown diameter is established. As transposition of the independent variable is involved error creeps in. Bonner (1964) used two variables e.g. crown diameter and tree height with good result. In large scale photos Sayn-Wittgenstein and Aldred (1967) tested different variables and found that total tree height times the logarithm of crown area stood out as the most powerful variable. With the use of large scale photography (70mm) taken from helicopter, species identification and reliable measurement of tree variables such as height, crown diameter and crown count is posssible - thereby volume estimation of trees is improved (Aldrich, 1966; Lyons, 1966).

CHAPTER 3

STATISTICAL ASPECTS OF VOLUME PREDICTION MODELS

3.1 General

In many forest inventories using the volume equation approach, the statistical aspects and underlying assumptions have not received adequate consideration (FAO, 1973).

Models for tree volume estimation have generally been linear models, meaning that they are linear in the coefficients, although they may be nonlinear as far as variables are concerned (Freese, 1964). In nonlinear models coefficients are raised to other than first power, appear as exponents or are combined by other than addition or subtraction. In the case of nonlinear models, direct application of the least squares principle requires iterative solutions to systems of nonlinear equations and the statistical properites of the resulting estimates are not well established. Because of these difficulties nonlinear models have not been used much for tree volume estimation, to date, and will not be examined here.

3.2 Problems Arising With Least Squares Analysis

The least squares principle rests on a number of assumptions. If these assumptions are not valid, various problems arise in the precision or bias of the model as a predictor.

3.2.1 Specification of model

The first and foremost condition is the correct specification of the model so that it adequately describes the data. Normally this assumption is expected to be met in case of volume equations as most relationships may be adequately represented by polynomial expressions (Gerrard, 1966). However, specification errors may arise due to incorrect functional form, in omission of independent variables and inclusion of irrelevant variables. Specification error can only be detected through repeated observations of independent variable using estimates of true error variance. If the model is not correctly specified, estimated regression coefficients will have biased estimates of true coefficients.

Tree volume models in this respect do not pose any serious problem. The number of explanatory or independent variables is quite limited and with judicious selection, such errors can be avoided. A function is selected which best explains the behaviour of the data. However, additional residual analysis may be carried out to detect whether any independent variable, which is important in increasing the predictive power of the model, has been omitted. This involves plotting residuals against the additional independent variable and checking for any systematic variation with the level of this independent variable (Neter and Wasserman, 1974).

3.2.2 Randomness of Error

Least squares principles assume random normal distribution of the error term of the regression. Sample points should cluster more or less randomly around the regression line. This is possible only when the positive or negative value of one error term does not have any effect on that associated with the other. This is ensured if random samples are collected. The individual observed values of a dependent variable will deviate from the population mean of the dependent variable by random errors which will be distributed about zero.

The above requirement necessitates that data must be obtained from sample plots which are well distributed in a representative manner throughout the population to which the prediction will be applied. Random sampling is the best for location of plots and trees. In forestry, however, simple random sampling is often replaced by stratified, cluster or systematic sampling (Bruce and Schumacher, 1950; Spurr, 1952; Cochran, 1953). In such cases the confidence limit of the estimate changes. However, use of cluster sampling in place of a random one does not have a great effect on the regression coefficients obtained. It is quite unusual to find volume tables with confidence limits in foresty literature (Cunia, 1964), though they should have such limits.

Any departure from random sampling in selection of sample trees for felling measurement or for dendrometry of standing trees renders the assumption invalid. However, in the present study the number of sample observations is too small to be tested. Furthermore, the sampling methods for selection of trees in this case are known to be strictly random.

3.2.3 Multicollinearity

In regression analysis it is essential that independent variables should not be correlated, i.e. a change in value of one independent variable should not cause a change in another.

A high degree of intercorrelation present amongst the independent variables in a model is termed as multicollinearity. When constructing regression models, it is often necessary to separate the relative effects of various intercorrelated variables on the dependent variable. This effect is often complex and has not been much investigated nor reported in forestry literature. When independent variables are correlated, the regression coefficient of one depends on which other independent variables are included. The coefficient does not show the real effect of any particular variable but only a partial effect. In such a case, the effect of every independent variable has to be judged in the context of the other variables present. In the presence of multicollinearity, only an imprecise estimate of true regression coefficients are available and these may not have much explanatory value. In the presence of severe multicollinearity, predictions are to be limited within the range of observations. This is not possible in practice in the case of tree volume equations, where, predictions are to be made for other samples and beyond the range used in preparing the model.

When there is a high degree of multicollinearity or intercorrelation amongst the independent variables are suspected, a correlation matrix of independent variables and dependent variables should be obtained. This will clearly display which independent variables are highly correlated. To eliminate this problem, the correct choice of independent variables is necessary. In the presence of multicollinearity, only one variable amongst many may be retained in the model. Also alteration or combination of certain independent variables may be useful. In some cases, the addition of more observations may break the pattern of multicollinearity (Johnston, 1972).

Multicollinearity is likely to exist amongst measurements of d.o.b. and d.u.b., tree height and point height and sometimes with point height and bark thickness. Due to this, judicious selection of independent variables for inclusion in the model is necessary and only one, out of intercorrelated sets of the above variables can be considered.

3.2.4 Serial correlation and independence of error

The assumption of independence of the error term is very important. For the sample values, derivations of values of the dependent variable from the regression surface must be independent of each other; size and direction (positive or negative) of the error of one value should have no connection at all with those of other value of the sample (Freese, 1964). Errors may cease to be independent if repeated observations are made on a single unit and also if measurements in the sample are taken on clusters of trees within plots. Model inadequacy in the sense of necessary variables may also cause apparent serial correlation (Wonnacott and Wonnacott, 1970). In general, the presence of serial correlation will not cause bias in ordinary least squares regression estimates, but the precision of those estimates will be reduced (Pindyck and Rubinfeld 1976). Thus the standard error of regression coefficients may be underestimated making confidence intervals and t and F-tests inappropriate.

In case of large samples tests may be applied to computed residuals of which the most important test is the Von Neuman ratio, the ratio of mean square successive difference to variance. For small samples, Durbin and Watson's 'd' statistic based on the above ratio consists of determining whether or not the autocorrelation parameter (P) is zero. The inconclusive range of the test is a problem. Various authors have suggested modifications (Theil and Nagar, 1961) and new tests have been devised (Hensaw (Jr.), 1966 and Theil, 1971). However, these tests are quite intricate and require large number of observations. Durbin and Watson's test (1950, 51) therefore seems satisfactory for the present project. This test can also be supplemented by graphical examination of residuals plotted against time for time series data. If the errors are independent, the residuals are expected to fluctuate at random around base line zero.

When significant autocorrelation is detected, a model should be developed which recognises the autocorrelation among the error terms. Ezekiel and Fox (1959) suggested use of first difference in place of original values. Archer (1977) suggested random culling of data to reduce the effect of clustering in the case of significant serial correlation in tree volume data with the assumption that improving model sepcification was not effective in the removal of such error. He worked on tree volume data which were collected by cluster sampling where serial correlation was likely to exist, originating in nonindependence of error terms due to the effect of similar site and tree association. If the autocorrelation parameter (P) is known, the dependent variable may be transformed with this value by substituting the first order autoregressive model and using ordinary least squares regression. Often P is not known but it can be approximated through an iterative approach (Cochrane & Orcutt, 1949). This method is not always successful. The method of first difference, where the autocorrelation parameter is assumed to be 1, and the transformed variables are the simple first differneces, is a simpler approach and has been found to be quite effective in reducing autocorrelation (Neter and Wasserman, 1974).

In the present study sample data had been collected from different tree farms and the trees were selected at random. Similar site factors and other connections had hardly any effect. Furthermore, no successive observations of tree measurements on a single unit are involved. Hence the presence of serial correlation was ruled out and independence of error terms can be assumed to hold.

3.2.5 Homogeneity of Variance

The most important assumption is that the sample data must be from a population for which variance is homogenous i.e. in which variance of dependent variable about the regression surface should be the same for all combinations of independent variables (Freese, 1964; 1967). This may also be called an assumption of constant error variance or homoscedasticity and its absence is called heteroscedasticity. In the presence of heteroscedasticity, ordinary least squares estimation places more weight on observations which have large error variances compared to those with small error variances (Pindyck and Rubinfeld, 1976). In these circumstances, provided other assumptions are met, the regression coefficients can still be unbiased and consistent estimates, but the variances of the estimated parameters are no longer at a minimum (Neter and Wasserman, 1974).

Assumption of homogeneity of variance is mostly invalid in the case of tree volumes. Trees of small dimension show less variation in volume than those with large dimension. Thus, variation in volume about regression surface is unlikely to be homogenous.

Various tests for null hypothesis of homoscedasticity against the alternative hypothesis of heteroscedasticity exist. The possibility of testing the hypothesis depends on the nature of the sample data. Three older tests viz, Hartley's Maximum F-ratio test (1950), Cochran's test (1941) and Bartlett's test (1937) can be mentioned where data is partitioned over the range of the independent variable suspected to be the source of variation. Two former tests are applicable if the number of observations for which variance is calculated is same for each cell. All these three tests are sensitive to mild departure from normality and are not robust (Sokal and Rohlf, 1969). Sokal and Rohlf mentioned that Bartlett's test is very sensitive to departure from normality and may sometimes indicate nonnormality rather than heteroscedasticity and so many statisticians do not recommend it. Other more powerful tests had been developed in the recent past which are suitable for smaller number of observations. Goldfeld and Quandt (1965) reported two exact tests - one parametric and other non-parametric for testing homoscedasticity. The parametric test has been discussed by various authors (Johnston, 1972; Pindyck and Rubinfeld 1976 and others). The power of the test depends on the number of central observations removed and the nature of sample. When observations with equal variances are removed, the power of the test improves. The second test is non-parametric and not much used. Glejser (1969) proposed an efficient test in which absolute values of least squares residuals are regressed on some function of the independent variable with which variance is associated. However, this test has certain limitations and Johnston (1972) recommended that the Goldfeld-Quandt test be used instead. Thus the latter test will be used in this study.

A graphical plot of residuals against fitted values of dependent variable is very effective in examining the homogeneity of error variance where the Goldfeld-Quandt test is not warranted.

3.3 Use of Transformations and Weighting

3.3.1 Transformations

Transformation of variables often helps to eliminate heteroscedasticity (Freese, 1964). Although the basic purpose of transformation of one or both variables in ordinary least square regression is to linearise the regression function, it often also stablises the error variance. Logarithmic transformations are most frequently used (Draper and Smith, 1966). In cases where variance increase with the increasing value of the dependent variable, both variables may be transformed to logarithms. Meyer (1953) recommended such logarithmic volume models. Silva (1976) obtained the best results using double logarithmic equations in Brazil. However, Cunia

(1964) pointed out that by taking logarithms, estimation of arithmatic mean is replaced by geometric mean and so estimation is biased.

3.3.2 Weighting

In correcting for heteroscedasticity observation with small variance should be allotted more weight than the observation with large variance. The weights used should be the inverse of the variance of the residuals about the regression surface (Freese, 1964; Cunia, 1964). In practice this corresponds closely to the variances of the dependent variable along its range. From a test of 9 volume equations Honer (1965) showed that weighted regression functions and those developed from suitable transformation of variables were superior to 3 unweighted models. Munro (1964) also found that precision of tree volume estimate was much improved by weighting. Gibson and Webb (1968) however, suggested that the method of weighting may not be crucial for developing a tree volume model. They suggested that partitioned multiple regressions were as efficient as weighted regressions for the same model and that the best estimates of larger volume classes could be obtained by the former method. But such partitioning of data is not statistically efficient as population is subdivided into segments containing much smaller numbers of observations. Error variance estimation is therefore less efficient compared to that obtained from a weighted model.

For the application of weighted least squares regression, information on the relative magnitude of the error variance must be known. If the variance is known to be proportional to one of the independent variables, or its square, the weight could be inverse of the independent variable or inverse of its square as the case may be. Knowledge of error variance may be available *a priori*. Where the relationship is not known *a priori*, empirical studies of the trends in error variance must be carried out. Generally, in tree volume studies error variance increases with increasing height or diameter (Gerrard, 1966 and Leech, 1973).

Commonly the weighting functions used in volume equations have expressed error variance as a function of D^2H. However, Gerrard (1966) found that the above relation did not hold good for his data on Canadian species of trees and he used a logarithmic form of variance as the dependent variable with diameter (D) and height (H as independent variables i.e. Log (Variance) = a + bD + cH. Munro (1964) studied weighting functions and found that variance of tree volume was directly related to $(D^2H)^2$ as did Gedney and Johnson (1959), Gibson and Webb (1968), Schimitt and Bower (1970), Burley *et al.* (1972) and Smalley (1973). FAO (1973) recommended drawing up of a list of most significant weighted expression of independent variables i.e. $1/(d.b.h.)^2$ and to apply multiple stepwise regression. Moser and Beers (1969) used an exponential relationship between variance and D^2H. Leech (1973) used a function of the form Log (Variance) = a + b Log (D) + c Log (H) after testing 7 variance estimation models. He found logarithmic models superior to models with variance as dependent variable and D^2H, $(D^2H)^2$, D and H as independent variables. Archer (1977) investigated the relationship of variance of volume within D^2H classes against mean D^2H for each class. The results were inconclusive due to the heterogenous nature of data. He therefore used $1/(D^2H)^2$ as weighting factor for correcting heteroscedasticity.

Weighting can be effected by introducing the inverse of the error variance function into the moment matrix of the regression or by transforming each observation by multiplying all variables concerned, including the implicit unity variable for the intercept term by the inverse of the square root of the error variance. Both approaches are equivalent (Furnival, 1961; Freese, 1964).

Cunia (1964) suggested the use of successive sets of weighting function through iteration till no changes are obtained in either regression coefficients or variance estimates. However, in many cases it has been found that one iteration is sufficient. Small changes in weighting function do not have any significant effect on estimates of coefficients and standard error. Thus in this study, only one weighting function was used.

3.4 Normality

Most statistical tests of regression estimates assume that error terms are normally distributed, although this assumption is not essential for unbiased estimates of the coefficients themselves. Non-normality does not generally cause serious problems (Sokal and Rohlf, 1969). Major deviations from normality, with highly skewed distributions may have some effect on the confidence limits, t-test and F-tests. Johnston (1972) and Sokal and Rohlf (1969) recommended that the normality of residuals should be tested. Normality of error terms can be studied by graphical examination of residuals through constructing a plot of residuals on normal probability paper. The plot should be linear over all the expressed values. Alternatively a Kolmogrov-Smirnov test, the third central moment (g_1) and fourth central moment (g_2) tests or other tests can be employed for the purpose. While these tests are better than graphical techniques, they are really only suited to cases where the number of observations exceed 200 (Snedecor and Cochran, 1967). Shapiro et al. (1968) most comprehensively investigated 9 different mehtods of testing normality and they found that the W statistic (Shapiro and Wilk, 1965) provided a superior measure of nonnormality. A combination of standard tests of kurtosis and skewness was found to be the second best. However, the former involves complicated computations and is not generally used. In the case of present study the number of observations is too small for any formal test of normality. So reliance had to be placed in the simple graphical plot of residuals on normal probability paper.

3.5 Grouping of Species

While developing tree volume models it is not generally possible to construct individual models for each and every species present in the area inventoried. This poses a problem in forests with a large number of species differing in economic value. Various approaches can be followed in such a situation. Volume models can be separately developed for relatively more important species and tree species of less commercial value or infrequent occurrence may be grouped together as "other species" and a common omnibus model developed for them. Different species can also be grouped into homogenous classes on the basis of general tree form and a separate model developed for each group. FAO (1973) suggested several methods for grouping of tree species : (1) by comparison of scatter diagrams of important parameters of each species; (ii) by covariance analysis; and (iii) by the method of automatic classification through a multivariate analysis.

Bowling (1951) compiled a volume table for regrowth eucalypts in Tasmania and tested for difference between species, localities and crown class groups. Very little difference was observed. In some other cases (in the U.S.A.) composite volume models needed percentage corrections for individual species (Spurr, 1952; Gevorkiantz and Olsen, 1955). In a covariance analysis of data pertaining to 3 different altitudinal classes of *Pinus caribaea* var. *caribaea*, it was found that these classes were not significantly different (Burley et al., 1972).

In the tropical forests grouping studies are of greater significance. Shield (1965) developed composite volume tables for Papua New Guinea rain forest species. In British Guiana various species had been grouped into taper index classes which were strongly correlated with species (Hegyi, 1965). Nash (1973) studied objective grouping possibilities of 20 species of Surinam forests. Hindley (1973) used comparison of scatter diagrams for rapid grouping studies for a large number of tropical rainforest tree species in Sarawak. Only one species stood out significantly. In an inventory of moist tropical forests of eastern India, 9 separate models for very important species and an omnibus model for large number of less important species has been developed (P.I.S.F.R., 1976).

Strictly objective comparisons of two or more regression equations which have been fitted to different samples can be done by covariance analysis and this is described by Snedecor and Cochran (1967). Weighted covariance analysis with the use of dummy variables suitable for use in tree volume estimation studies has been described by Cunia (1973). This method is more general and flexible. It offers larger degrees of freedom (D.F.) for more efficient testing of hypotheses. It was therefore, used in the present study.

CHAPTER 4

TREE VOLUME DATA

4.1 Obtaining Tree Volume Data

4.1.1 Selection of sample trees

The main use of tree volume prediction model is in the forest inventory. Volume model development is carried out in a separate phase during the course of inventory work. Selection of trees, either standing or felled for measurement, should be statistically acceptable and the sample must be representative. Sample trees should be selected objectively with all variations in site, locality, age, size classes, relative abundance of species and other growth conditions being represented. Sample trees should not be rejected for felling and measurement if found to be defective. It is not uncommon to collect tree data from areas of commercial exploitation e.g. from windthrow areas, open areas close to the road and even in sawmills. Sandrasegaran (1969) used thinned trees as samples for constructing volume tables for manmade forests. Such samples are not representative and volume models developed form such samples have restricted application.

In Surinam survey, data had been collected from felled trees in as many diameter class as possible (Nash, 1973). In tropical forests, problems of accessibility, ruggedness of terrain, safety of crews, unfavourable conditions for felling and measurement, logistic considerations are very important and it is generally impossible to cover all potential variations within the population. Due to economic and logistic reasons sample tree data have to be collected often through concentrated logging operations. Hindley (1973), however, found that volume models developed from such data have inherent limitations. Burley et al. (1972) and Turner (1972) reported most possible objective methods of sample tree selection under prevailing circumstances. In several tropical forest inventories in eastern India, sample plots were selected at random and sub-samples of trees were felled after total enumeration in the plot (P.I.S.F.R., 1976). Such random sampling was also carried out by Wong (1966) in Malaysia. The author found this method logistically very difficult and had difficulty in achieving the target number of sample trees in various size classes. Yet, a maximum degree of objective selection was possible through such a method. FAO (1973) also emphasised that geographic distribution of the plots, from which sample trees are selected, should preferably be sampled objectively, either systematic or random.

4.1.1.1 *Number of sample trees*

The number of sample trees required varies with the precision desired in volume prediction and no set rule can be laid down. Representative distribution of sample trees per species or per diameter class is the most desired way, but the stand table of diameter distribution in unevenaged forest (reverse 'J' pattern) presents problems. Trees with larger diameter are less frequent but such trees are the most important ones, as their contribution to stand volume is high. Equal number of sample trees in all diameter classes will increase the error of estimation in larger trees. This may be remedied by proportioning numbers of sample trees per diameter class with relative volume in each class. It is found that the number of sample trees attained closely approximates Neyman's or optimum allocation (FAO, 1973). The cardinal principle is, the more the number of judiciously selected trees, the better is the precision of estimate. However, range of height, diameter, variability in species, site factors, forest types etc control the actual number.

4.1.2 Representativeness of sample

4.1.2.1 *Variation within and between species*

Generally it is found that different species of trees have different volumes for a particular dimension. Often the locality, growth pattern and site conditions apparently do not affect the total volume to justify more than one model for a given species (Spurr, 1952). Bowling (1951) worked on regrowth eucalypt forests of Tasmania and tests of different species groups showed very little difference between species, localities and site qualities. Gibson and Webb (1968) also applied one model for various regrowth eucalypt species of Victoria. Variation between different species in the tropics is quite significant and such forests usually have large numbers of species per unit area, Hegyi (1965) reports 200 on 80 ha. Pronounced differences in volume exists between shade tolerant and intolerant species. Inadequate botanical knowledge and similar appearance of trees of different species accentuate the problem. Very little work has been done in tropical forests with respect to variation in volume of trees within and between species.

4.1.2.2 *Variations due to site*

Different local conditions prevailing in various sites, supporting tree population affect the growth, form and total volume of trees. Trees on upland, ridge, swamp, lowland or rocky exposed sites have different growth rates and may also show wide variations in other parameters. Trees of the same diameter and height may differ in volume due to differences in shape and taper. The extent of such differences and their correlation with species, provenance and environment should be established (Carron, 1968). Trees on ridges have less height for a d.b.h. class compared to trees of other sites. Most taper is exhibited generally by trees on ridges and upland while lowland trees have less. Graphs of bark thickness against d.b.h. indicate a wide degree of variability in swamp, upland and lowland sites. Barnard et al. (1973) tested the difference in Girard form class measurements within species located in different places extending over 5 states in the U.S.A. and no significant differences were found. Limited research regarding variation in volume due to site and locality in the tropics have been reported (Wong, 1966). Thus, it is essential that variation patterns are studied before collecting sample tree data for developing volume models.

4.1.3 Measurement procedure

The extent to which volume prediction models may be validly applied to forests during inventory depends to a great extent on the methods used for measuring sample trees. Diameter, girth of sectional area of standing trees, should be measured at a fixed height above ground level e.g. breast height (b.h.). Such a point or any alteration of it due to various contingencies should be properly defined. Girth or diameter is generally measured, though the choice depends on local practice rather than by objective choice (Carron, 1968). For obvious geometric reasons measurement of girth of trees with cross section other than circular, leads to over estimation. In such cases the average of two diameter measurements at right angles gives more precise estimates. In Australia girth measurement is more common, but in tropical hardwood forests, the shape of the bole is generally irregular and calipers are commonly used for diameter measurement. In such forests, due to presence of buttresses and fluting, measurements are often made at higher points. All deviations from measurements at representative points have to be objectively carried out. Under ideal conditions, assuming the tree is straight and vertical, height is measured as the distance from ground level to the highest point of the tree. This is well suited to conifers but for eucalypts and tropical hardwoods some measurement of the bole is of practical interest. Similar principles, methods and instruments for measurement of tree variables should be used both for sample trees used in the volume model and the trees of forests to which the model is being applied.

4.1.3.1 *Collection of data for developing a volume model - object and general procedure.*

Data collection for volume tables involves 1) selection of a judicious sampling method, 2) deciding on sample size and the method of distribution and 3) procedure of measuring tree volume. Various aspects of sampling have been dealt with in statistical tests (Sukhatme, 1954; Cochran, 1953). Works by various authors on sample tree selection for volume study were mentioned earlier. Methods of measuring sample trees can be grouped into two categories, viz 1) measuring standing trees with optical dendrometers (Barr and Stroud, telerelascope etc). 2) measurement after felling of trees. Before measurements are taken, the part of the tree for which volume will be measured is to be defined clearly. Generally, the tree volume to be measured corresponds to a specified section of the stem, normally up to the merchantable limit, over bark or under bark. If the utilisation is very intensive, branch volume also assumes great importance and similar principles of measurement are extended to merchantable wood from branches. For many hardwood trees of both tropical and temperate countries, branch volume extends up to as high as 30 percent of total volume and considerable portion is utilisable. In forest inventories of India branchwood is considered in estimating potentially available growing stock.

Methods of direct determination of tree volume have been dealt with by Husch et al. (1972). Felling of trees may not be essential in all cases. Tree volume can be efficiently estimated from measurements offered by optical dendrometry and use of computer programme (Grosenbaugh, 1963). This method is most suited to trees of excurrent form with low branch volume. These measurements are limited to over bark, bark factors being used for calculation of under bark volume. In Australia the Barr and Stroud Dendrometer has been used in research as described by van Schie (1967) in measuring standing sample trees in regrowth forests of southern Tasmania. In tropical and temperate mixed hardwood forests, inventory projects normally assess quality with growing stock appraisal. In such cases sample trees are felled for satisfactory measurement of external and internal defects. Dowden, (1967) found that for eucalypt resource assessment in Victoria, non-destructive methods of quality assessment were not reliable. However, Hindley (1973) advocated non-destructive quality assessment by boring for decay as excellent results were obtained in Malaysian forests.

Methods of measuring merchantable bole volume or total volume of felled trees include 1) formula method applied to sections of trees; 2) graphical method and 3) method of water displacement (Husch et al., 1972). In method (1) the careful selection of formulae is required. Young et al. (1967) found that in a comparative study, Newton's formula furnished most accurate results, followed by Huber's formula and Smalian's formula. Similar results were reported by Miller (1959).

Accuracy of Newton's formula increases as short sections are measured. FAO (1973) highly recommended use of Newton's formula if the number of logs per tree is small. Individual tree taper measurements are essential in using the graphical method. The third method is the most accurate. The volume of any part of the tree is found by submerging it in a xylometer in which the displacement of water is accurately read. This method has limited applicability but various workers used this for comparative studies (Young et al., 1967; Dargavel and Ditchburne, 1971).

The number and length of section to be made for each sample tree depends upon the type of product. While it is necessary to have small sections for ensuring maximum accuracy, minimum utilisable size and economic aspects are also to be considered. In India, the author had tree sections of 2.5m length for sawlog and plylog volume estimation. This facilitates both collection of data with end measurements of logs by using calipers as well as computation of volume using Smalian's formula. Small wood (from 20cm d.u.b. to 4cm d.u.b.) was cut to convenient lenghts and Huber's formula was used for its volume computation. In tropical hardwoods Hegyi (1965) showed that no significant difference in volume estimates exists when a tree was measured in three or more sections. To avoid bias in the measurement of the butt section, it should be of shorter length. The positions of measurements, marked with an axe cut, should also be fixed, so that consistency in measurement by field crew is ensured.

4.1.4 Data processing

In data processing the basic recorded data are arranged into a suitable form for subsequent processing. The arrangement of data in a computer readable form is called data capture. By and large, modern tree volume prediction models are being developed through analysis undertaken with the help of Electronic Data Processing (EDP) for greater efficiency. In most of the developing countries the use of computers is increasing for forest inventory data processing. Standard size punch cards are the common mode for data capture, as punch card readers are available in all computer centres. In developed countries, demand run systems with terminal facilities are commonly available where data can be directly transferred to the discs etc. After initial data capture, editing ensures a clear data file free of various errors and inconsistencies. This consists of sorting and error detection and subsequent checking of extreme values. Inconsistencies are then corrected in collaboration with the field crew leader. Such rigorous checking constitutes an essential part of the construction of volume models as inconsistencies and errors in data affect the accuracy of the volume estimation. Unwin and Bowling (1951) described manual and graphical checking of data for tree volume tables in Tasmania. Carron (1968) suggested graphical plotting of sectional area at different points along the stem to arrive at a reasonable trend in stem profile. Various computer programmes developed for the calculation of tree volumes from sample tree data provide for checking of errors in the data set. Before checking for reasonableness of data, acceptable limits of variation are finalised for the population. As tree volume is approximately proportional to D^2H, the volume prediction is highly sensitive to errors in d.b.h. measurement. As gross volume under bark is often desired, checking the measurement of bark thickness should also be carried out. In a highly variable population, wide variations in measurements should be expected.

Extremely unreliable observations in the data set will be considered as outliers, and when present, they pose serious problems in regression analysis. The detection of outliers in data is very difficult and this can be accomplished best by residual plotting. Due to their presence the fitted line is pulled disproportionately towards them. However, outlier data should be treated with caution. Dixon and Massey (1957) discussed this problem. An observation duly suspected as being outlier should be removed only if errors in recording or mistakes in measurements are established (Neter and Wasserman, 1974). The rejection of such outliers without valid reason is not acceptable to statistical principle which assumes that when an infinite population is sampled such measurements are theoretically possible (Nash, 1965).

Volume computation of felled trees is generally performed using a suitable computer programme. Various authors used Smalian's formula for each section above the stump (Stage et al., 1968). A programme developed for forest inventory data processing in India used Smalian's formula for lower sections and Huber's formula for smallwood sections.

4.2 Study Data

4.2.1 Study area

The present study on tree volume equations is based on data from A.P.M. Forests Pty Ltd, Gippsland, Victoria. The forests are located in Victoria and broadly divided into 3 groups viz,

1) Exotics plantation (*Pinus radiata* D. Don)

2) Ash type eucalypt forests or mountain forests comprising mainly of *Eucalyptus regnans* (F. Muell), *E. delegatensis* (R.T. Baker), and *E. nitens* (Deane et Maiden).

3) The mixed species eucalypts, which consist of the following species are:

(i) *Eucalyptus globulus* (Labill)
(ii) *Eucalyptus obliqua* (L. Herit)
(iii) *Eucalyptus sieberi* (L. Johnson)
(iv) *Eucalyptus radiata* (Sieb.)
(v) *Eucalyptus cypellocarpa* (L. Johnson)
(vi) *Eucalyptus baxteri* (Benth)
(vii) *Eucalyptus muelleriana* (Howitt)
(viii) *Eucalyptus globoidea* (Blakely)
(ix) *Eucalyptus consideniana* (Maiden)

The forests vary in stocking. This wide range is due to differences in climate and soil moisture which, in turn are due to variation in elevation (from sea level to 1200 metres A.S.L.) over the study area. These forests supply raw material to a pulpmill, sawmill and a particle board factory. Pulping quality varies with species and age - four categories viz, A, D, C & D are recognised. The current demand is high and these forests, mainly the radiata pine plantations have to support a very large expansion of the pulp mill. As the company planned to use smaller size eucalypts in production of pulp, older plantations of eucalypts and regrowth mixed eucalypt forests are assuming greater importance.

4.2.2 Selection of sample trees

This section discusses the collection of sample tree data in mixed regrowth eucalypt forests in Gippsland, Victoria for the construction of a volume prediction model. The sample trees measured in 1968 were 12 - 25 years old and belonged to sample series No. 5, 16 and 17. As it was necessary to measure sample tree volume with the minimum error, it was decided that measurements should be made on felled trees only. In anticipation of the future use of the data, additional over bark measurements on the sample trees were made and these measurements were retained in "Sample Tree Library".

The data was collected to construct a single tree volume regression for the most important eucalypt species on company property in Gippsland. Stands of regrowth in which the trees are larger than 12.7cm d.b.h. and younger than 60 years (saplings and poles) had been delineated on a map. Principal areas of sampling are Jeeralang/callignees Tree farms type mapped area, Silver Creek Tree farm assessed area, Moondarra-Boola Tree farm assessed area, Boola working plan area and oldest cut over areas, in Erica Forest Division. The principal eucalypt species are as follows:-

(i)	*Eucalyptus sieberi*	(Silver-top ash)
(ii)	*Eucalyptus obliqua*	(Messmate stringy-bark)
(iii)	*Eucalyptus globulus/St. Johnii*	(Southern blue gum)
(iv)	*Eucalyptus cypellocarpa*	(Mountain grey gum)
(v)	*Eucalyptus muelleriana*	(Yellow stringy-bark)

The diameter distribution of the population was first estimated from the data already available. The number of trees to be selected for each of these species was limited to 40 - 50. Ten locations were pickd at random for messmate stringy-bark, silver-top ash and five each for the rest. At each location 4 - 8 trees were selected. At each location the nearest tree above the average diameter and nearest tree below the average diameter were selected. Trees of grossly abnormal form were not selected. The diameter distribution of the first 40 trees selected was compared with that of the population. It is necessary to ensure that each diameter class in the sample is represented in the same proportion as its occurrence in the population. The number and sizes of further sample trees required to achieve this was determined. The additional trees were obtained by selecting further points at random in the plantation, at each of which the two trees nearest to the required sizes were selected. The number of such additional trees was restricted to ten.

4.2.3 Measurement procedure

4.2.3.1 *Determination of breast height*

The Company has adopted the following definition of breast height (Dargavel, 1969):

"a) The measurement point must be a clear point. A clear point is defined as one which is clear of bumps by being more than 10cm above the whorl and more than 15cm below the whorl.

b) The measurement point must be at 1.3m vertically above the mineral soil level measured on the uphill side of the tree.

c) If a clear point cannot be located at 1.3m level, the nearest single clear point will be taken within 23cm up and down the stem.

d) If a single clear point cannot be located within 23cm of the 1.3m point, that is between 1.07m and 1.53m, two clear points will be taken spaced as near to equidistantly above and below the 1.3m point as possible. The arithmetic mean of the two diameters will be taken as the diameter of the tree.

e) Multiple stems which fork below the breast height point are treated as separate trees while those which fork above the point are treated as a single tree."

4.2.3.2 *Stem measurement*

The A.P.M.F. method was followed for stem measurement. D.b.h. o.b. was measured on the standing tree to the nearest 2.5mm with the circumference tape calibrated to read diameter. Breast height was marked with a light axe cut. Further measurement points were marked with light axe cuts after the sample tree was felled. The measurement points were selected to be at stump height and at "clear points" at the stem at approximately 61cm and 1.53m above the ground level. Further points were selected at about 1.53 intervals up to the merchantable height or point height. The point diameter (u.b.) varied from 3.2cm to 11.4cm. Additional measurement points were marked if sudden changes occurred.

The stump height was pre-determined before felling and this made it possible to record an accurate measurement of d.o.b. and d.u.b. before the stump was damaged by felling. The height of the selected measurement points were measured with a tape stretched along the tree. The height of the points at which first green branch and first whorl appeared were also recorded. Over bark diameters at each point were measured and, at each point a band of bark was carefully removed and under bark diameters measured. All such measurements were carried out with a tape and these were to nearest 2.5mm. These diameter measurements were converted to cross-sectional areas and plotted on graph paper with axes of cross-sectional area and height. This plotting was examined before field party left the sample tree. If any abnormality was noticed, repeat measurements were made and abnormalities, if confirmed, were recorded on the field graph plot sheet.

4.2.3.2 *Other parameters measured*

Crown diameters were measured with a plumb line and tape. Crown class was assessed and recorded in either of the following 5 classes: 1) Good crown 2) Defective 3) Abnormal 4) Sparse foliage and 5) Deformed.

Tree form was assessed and each tree had been grouped in either of the following types: 1) Good, 2) Defective, 3) Heavy branching, 4) Sweep, 5) Broken, 6) Bumps, 7) Spiral, and 8) Other.

Stump height, maximum d.b.h.o.b. and d.b.h.u.b., position of major branches and effective merchantable height were also recorded.

4.2.4 Data processing

Sample tree data was stored on magnetic tape in the computer of A.P.M. Pty Ltd. The data set had been checked by an edit programme for correct entries in various columns viz, sample series, tree number, species, ownership, crown class, DBH (OB), DBH (UB), stump height, point height, etc. Faulty code, anomalous values due to wrong entries or error in measurment were corrected in consultation with the field crew. With this preliminary checking, the possibility of major errors occuring in the data was avoided.

All felled tree measurements were plotted on a graph as already mentioned and the tree profile was inspected for sensibility. Sample tree measurements, which showed a wide range of variation after thorough checks, were not considered as abnormal. These were considered as part of the population with wider variabilities. During the analysis of data no outlier was rejected.

Gross merchantable tree volume (u.b.) up to the point diameter (u.b.), was calculated by a computer programme using Smalian's formula.

4.2.5 Techniques Used

A computer package - GLIM 3 developed by the Royal Statistical Society, London was used for fitting of regression models. GLIM is a programme, designed to facilitate the fitting of generalised linear interacting models. In the Univac 1100 computer of the Australian National University interactive mode is available allowing on-the-spot correction of errors. This enabled exploratory studies of data by trying various transformations, sub-division of data, adding and deletion of data sets. Graphical studies of relationships of variables and residuals are possible in this package. GLIM provides facilities of iterative weighted least square analysis and regression can also be fitted through the origin. Various simple statistics can be calculated with this programme but the plotting of weighted residuals is not possible.

4.2.6 Choice of level of significance to be used

In model development studies, a level of significance has to be assigned for significance tests. Two types of errors are common in statistical analysis when hypothesis testing is carried out. When a true null hypothesis is rejected type I errors occur and when a false null hypothesis is accepted type II errors occur (Sokal and Rohlf, 1969). These two types of errors should be kept at a minimum level. Also a balance between probabilities of their occurrence is to be maintained as when one is reduced, the other increases.

The data set contains 125 observations, grouped into 5 species. As measurement of volume was likely to be only moderately precise, a lower significance level was considered suitable. The level chosen was $P = .05$.

CHAPTER 5

ANALYSIS OF DATA

This chapter deals solely with the results of analysis of the models based on diameter over bark and tree height. Results for models utilising diameter under bark or merchantable height follow a similar pattern and are summarised in Chapter 6.

5.1 Graphical Studies on Grouping

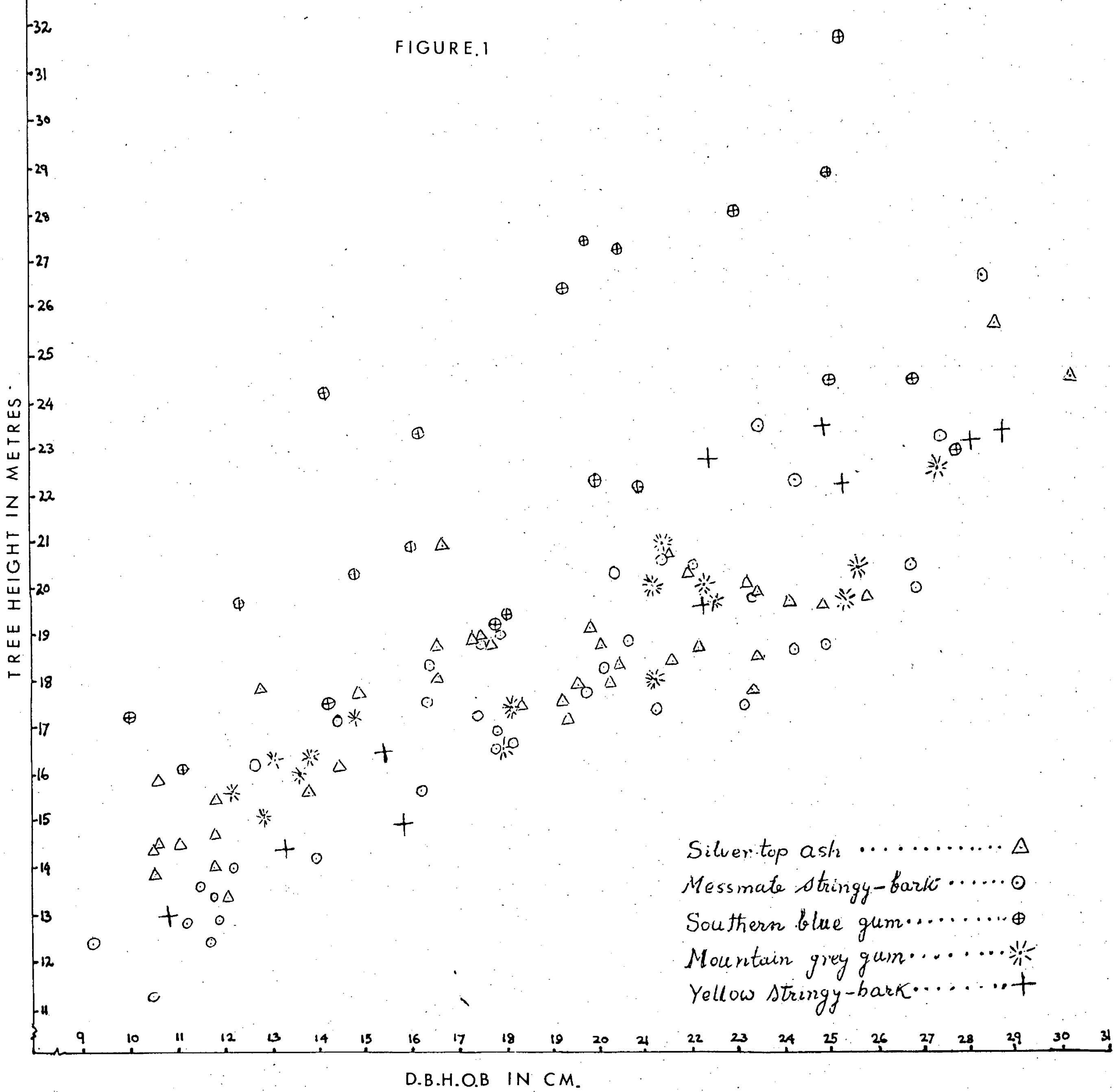
In this study 5 data sets pertaining to 5 different species of eucalypts were examined graphically to see whether the species could be grouped for the purpose of volume prediction.

Data pertaining to d.b.h.o.b. were plotted against height and are shown in Figure 1. In the case of southern blue gum the scatter of points was found to be markedly different having greater height for a given diameter than the other species. The scatter of points could not be effectively separated for the other groups.

Another significant characteristic of these species was bark thickness. Messmate, silver-top ash and yellow stringy-bark have rough stringy-barks of similar nature. Southern blue gum and mountain grey gum have smooth, dense "gum" type bark. D.b.h.o.b. values were plotted against twice the bark thickness (2BT) for this data as shown in Figure 2. While messmate did not show any specific

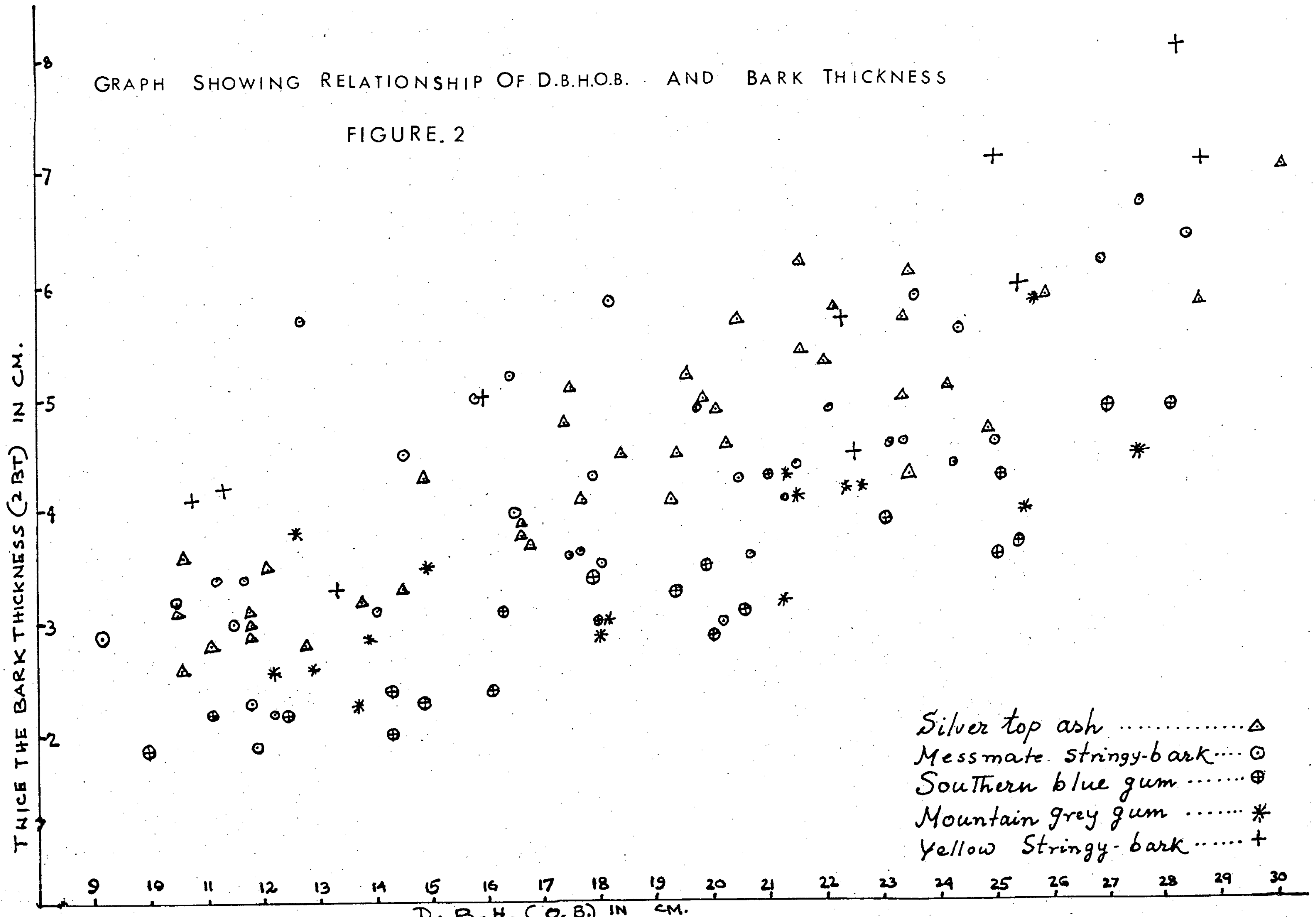
GRAPH SHOWING RELATIONSHIP OF HEIGHT AND DIAMETER

FIGURE.1



GRAPH SHOWING RELATIONSHIP OF D.B.H.O.B. AND BARK THICKNESS

FIGURE. 2



trend, the spread overlapping other species, silver-top ash and yellow stringy-bark showed marked differences compared to the scatter for mountain grey gum and southern blue gum. Thus graphical analysis indicated possibilities of grouping smooth-bark and rough-bark eucalypts into distinct sets.

5.2 Fitting of Unweighted Models

Various authors have developed models including as many as 10 terms to explain the relationship between volume and other variables. Detailed justification for such complicated models is generally lacking in the forestry literature. Although no single model has been identified in the literature as being suitable for all species under all conditions, the combined variable (D^2H) term is included in most. Furthermore, Honer (1965) studied the relative usefulness of 15 independent variables of various combinations of d.b.h. and height in explaining variation in tree volume. He showed that the D^2H term accounted for 97 percent of the total variation of the dependent variable, whereas the inclusion of any second term in D or H or both only accounted for an additional 0.2 percent of total variation at best.

Based on previous research, a list of 16 different forms was drawn up as shown in Table 5.1: most include the combined variable term.

Each of these models was fitted in turn to the data for all species pooled together. The results of these regressions are summarised in Table 5.2. High multiple determination coefficient (R^2) was obtained for all the models (above .967). However, only 6

TABLE 5.1

VOLUME ESTIMATION MODELS (UNWEIGHTED)

(OVER BARK)*

		Equation
1.	$v = b_0 + b_1H + b_2D + b_3D^2 + b_4H^2 + b_5DH$	5.1
2.	$v = b_0 + b_1H + b_2D^2 + b_3D^2H$	5.2
3.	$v = b_0 + b_1D + b_2DH + b_3D^2 + b_4D^2H$	5.3
4.	$v = b_0 + b_1D^2H$	5.4
5.	$v = b_0 + b_1H + b_2D^2H + b_3H^2 + b_4D^2H^2$	5.5
6.	$v = b_0 + b_1D^2H + b_2D^3$	5.6
7.	$v = b_0 + b_1D^2H + b_2D^2 + b_3D^3$	5.7
8.	$v = b_0 + b_1D + b_2H + b_3D^2H + b_4H^2 + b_5DH^2 + b_6D^3 + b_7H^3 + b_8D^2$	5.8
9.	$v = b_0 + b_1D + b_2H + b_3DH + b_4D^2 + b_5D^2H$	5.9
10.	$v = b_0 + b_1D^2H + b_2D^3H$	5.10
11.	$v = b_0 + b_1H + b_2D^2 + b_3H^2 + b_4D^2H$	5.11
12.	$v = b_0 + b_1D + b_2H + b_3DH + b_4D^2 + b_5D^2H + b_6H^2 + b_7DH^2$	5.12
13.	$v = b_0 + b_1D + b_2H + b_3DH + b_4D^2 + b_5D^2H + b_6H^2 + b_7DH^2 + b_8D^2H^2 + b_9D^3 + b_{10}H^3$	5.13
14.	$v = b_0 + b_1D^2H + b_2(D^2H)^2$	5.14
15.	$v = b_0 + b_1D^2H + b_2D^3H + b_3(D^2H)^2$	5.15
16.	$v = b_0 + b_1D^2H + b_2D^3 + b_3(D^2H)^2$	5.16

* D = Diameter at breast height; H = tree height.

TABLE 5.2

VOLUME ESTIMATION MODELS (UNWEIGHTED)

SUMMARY OF REGRESSION STATISTICS (OVER BARK)

	SS	DF
Total	1.8180	124

EQUATION	Regression		R^2 Adjusted	SIGNIFICANCE OF COEFFICIENTS
	SS	DF		
5.1	1.7796	5	.978	H,D,D^2,H^2
5.2	1.7717	3	.9734	H,D^2H
5.3	1.7722	4	.9742	DH
5.4	1.7590	1	.9668	D^2H *
5.5	1.7776	4	.9770	H,D^2H,H^2,D^2H^2 *
5.6	1.7700	2	.974	D^2H,D^3 *
5.7	1.7701	3	.9734	D^3
5.8	1.7828	8	.9797	None
5.9	1.7723	5	.9740	None
5.10	1.7636	2	.9695	D^2H,D^3H *
5.11	1.7792	4	.9783	H,D^2,H^2,D^2H *
5.12	1.7724	7	.9788	D,H,DH,DH^2
5.13	1.7831	10	.9793	None
5.14	1.7592	2	.9672	D^2H
5.15	1.7719	3	.9740	$D^2H,D^3H,(D^2H)^2$ *
5.16	1.7700	3	.9724	D^2H, D^3

* All the coefficients in the model are significantly different from zero.

of the models in Table 5.2 had values of the coefficients and associated standard errors such that all were significantly different from zero at the 95 percent probability level.

5.3 Testing Assumptions

Examination of the scatter diagrams of the residuals against fitted values for the above models indicated that there was a marked trend for the variance of the residuals to increase as the predicted value of volume increased. However, it was not clear whether this property was common to all species or resulted from differences between the species. No other problems were apparent.

The data were subdivided by species into 5 groups as discussed in Chapter 4. As yellow stringy-bark and messmate are similar in nature and former had only 10 observations, these were combined to form Species Group 2. Silver-top ash, southern blue gum and mountain grey gum formed species group 1, 3 and 4 respectively.

Further regressions were fitted to each Species Group separately, but only for models 5.4 and 5.14 in Table 5.1. The scatter diagrams of residuals for both models and for each species indicated that heteroscedasticity was present in each. This was also confirmed by Goldfeld-Quandt test for homogeneity of variance. The results of the test are summarised in Table 5.3.

The data for the test for the first three species groups were rearranged in ascending order of D^2H values which were then divided into three parts with an equal number of observations in the first and the third parts. The central part in the case of groups 1, 2 and 3 consisted of 11, 11 and 5 observations respectively and these were

TABLE 5.3

HETEROSCEDASTICITY TEST

	S_2	S_1	$R=S_2/S_1$	DF	Critical F value
Species Group 1 Models:					
5.4	.00276	.00045	6.13	13	$F_{(05;13,13)} = 2.57$
5.14	.00181	.00044	4.11	12	$F_{(05;12,12)} = 2.69$
Species Group 2 Models:					
5.4	.00809	.00104	7.79	16	$F_{(05;16,16)} = 2.33$
5.14	.00714	.00104	6.87	15	$F_{(05;15,15)} = 2.41$
Species Group 3 Models:					
5.4	.00864	.00036	24.00	6	$F_{(05;6,6)} = 4.28$
5.14	.00829	.00014	59.21	5	$F_{(05;5,5)} = 5.05$
Species Group 4 Models:					
5.4	.00232	.00028	8.29	6	$F_{(05;6,6)} = 4.28$
5.14	.00216	.00027	8.00	5	$F_{(05;5,5)} = 5.05$

S_1 and S_2 are sum of squares of residuals from regressions based on relatively small and large values respectively.

then discarded and the remainder analysed in the test. For Species Group 4, however, there were not sufficient observations to enable this to be done. In this case the ordered set was halved and each half used in the analysis. For each species, and both models the variance of the residuals proved to be heterogenous at the 95 percent probability level.

In view of the nature of the data and the method of collection as stated in Chapter 3, no other test viz, tests for serial correlation and randomness, were considered necessary.

5.4 Weighting Functions

Since the data are heteroscedaceous, ordinary least squares regression was considered to be inadequate to estimate the regression coefficients. Various authors agree that a weighted least squares solution should be applied in such a case. Before the development of a suitable weighting function, a preliminary examination of the variance and its relation to D^2H values was conducted. Because Species Groups 3 and 4 had so few observations, but were both smooth-barked, it was decided to pool these. For each species group the data were ordered according to the value of D^2H, being the most important variable. Data sets for species 1 and 3/4 (pooled) were partitioned into 5 groups containing almost equal number of observations ordered on D^2H. Data for Species Group 2 had been divided into 6 classes as it had more observations. This procedure had the advantage that it ensured almost equal numbers in each group and, as it proved later, relatively uniform spacing in terms of the logarithms of the mean value of D^2H. For each group the variance of the volume was calculated. The results for the mean values of D^2H of each class and corresponding values of the variance of volume are furnished in the Table 5.4. Variance generally increases with increasing D^2H in all Species Groups.

In earlier research (e.g. Gerrard, 1966; Leech, 1973) as discussed in Chapter 3, it is found that the relation between the variance of volume and D^2H, in certain cases, may be exponential. Scatter plots of these variables supported the hypothesis that an exponential function relating variance to volume was appropriate after taking the logarithm to the base 10 of both variables. The exponential function was fitted to each data set and the coefficients were estimated through ordinary least squares analysis.

Analysis of covariance summarised in Table 5.5 showed that weighting functions of group 1, 2 and 3/4 (pooled) were not significantly different from one another. This showed that all the groups could be pooled to have the same weighting function.

The data for all the species had been combined again, ordered on D^2H values and divided into 15 classes of ordered observations of which 14 had equal numbers in each. This procedure is more efficient as it improved the dispersion of the observations along the variance function over that available from simply amalgamating the group mean observations as shown in Table 5.4. The data regarding mean $D^2H/1000$ and variance x 10000 have been summarised in Table 5.6. A simple linear regression was fitted to the logarithms to the base 10 of the values shown in Table 5.6. The resulting coefficients of variance estimation model from this, together with those for each of the earlier Species Groups have been furnished in Table 5.7. Results

TABLE 5.4

VARIANCE OF VOLUME AND MEAN OF D^2H CLASS

OVER BARK					
Species Group 1		Species Group 2		Species Group 3 & 4 Pooled	
$D^2H/1000$	Variance x 10000	$D^2H/1000$	Variance x 10000	$D^2H/1000$	Variance x 10000
1.8	.5	1.6	1.4	2.45	2.3
3.9	5.1	3.2	1.6	4.39	7.7
6.3	3.9	5.4	4.5	7.24	6.2
8.7	15.6	8.5	6.5	10.24	16.5
15.4	167.0	11.9	5.1	16.92	51.4
-	-	17.6	36.1	-	-

TABLE 5.5

COMPARISON OF VARIANCE FUNCTION
OF DIFFERENT SPECIES

[OVER BARK]

Between species 1 and species 2					
Regression line	ESS	DF	MSE	F	Remarks
Group 1	.3010	3			
Group 2	.2637	4			Critical value of $F_{2,7}$ at .05 P is 4.74
Pooled residuals	.5647	7	.0806		
Group 1 & 2	1.225	9			
(Single regression by pooling class means and variance of 1 & 2 together)				4.10	
Difference	.6603	2	.3301		Hence no significant difference exists

Between 1 & 2 combined and 3 & 4 combined					
Regression line	ESS	DF	MSE	F	Remarks
Group 1 & 2	1.2250	9			Critical value of $F_{2,12}$ at .05 P is 3.89
Group 3 & 4	.1101	3			
Pooled residuals	1.3351	12	.11126		
Group 1,2,3 & 4 combined (Single regression by pooling class means and variance)	1.4111	14		.341	Hence no significant difference exists.
Difference	.0760	2	.0380		

TABLE 5.6

VARIANCE OF VOLUME

AND

MEAN OF D^2H CLASS

pooled and regrouped data of Species Group 1,2,3 and 4	
Over bark	
$D^2H/1000$	Variance x 10000
1.5	.2
1.8	.5
2.3	.4
3.0	1.5
3.9	3.3
5.1	3.1
5.6	3.8
6.2	1.6
7.4	5.1
8.7	4.1
9.7	3.1
10.5	6.8
11.9	7.3
14.2	17.3
20.0	57.0

TABLE 5.7

VARIANCE ESTIMATION MODELS

(OVER BARK)

Group 1	$\log_{10}$ (Variance x 10000) = $-.9913 + 2.473 \log_{10} (D^2H/1000)$ (.3678) (.4490)
Group 2	$\log_{10}$ (Variance x 10000) = $-.2457 + 1.182 \log_{10} (D^2H/1000)$ (.2555) (.2974)
Group 3 & 4	$\log_{10}$ (Variance x 10000) = $-.2163 + 1.465 \log_{10} (D^2H/1000)$ (.2580) (.2946)
Groups 1,2,3 & 4 with data combined and regrouped	$\log_{10}$ (Variance x 10000) = $-.8676 + 1.751 \log_{10} (D^2H/1000)$ (.1538) (.1853)

indicated in this table further illustrate that there was no difference between the values of the slope coefficients, given the magnitude of the standard errors in brackets. Due to reasons aforesaid, the final estimates of the pooled and re-grouped data are more precise however. Thus, appropriate weighting functions were estimated to enable the heteroscedastic error terms to be transformed to a homogenous state.

5.5 Fitting Weighted Models

Models 5.8, 5.12 and 5.13 in Table 5.1 were not pursued further because they included so many independent variables, coefficients of most of which were not significantly different from zero. The remaining 13 models were fitted to the pooled data for all species using the weighting function as developed in the previous section. A brief summary of regression statistics have been shown in Table 5.8. Only 5 models had coefficients all of which were significantly different from zero. Nevertheless, the standard error (S.E.) of estimates of coefficients of weighted models were generally lower than those resulting from unweighted models, as would be expected. This also indicated that weighting had been effective. Scatter plots of residuals against fitted values of the dependent variable showed uniform spread at all levels suggesting that variance of error term has now become homogenous. Comparative scatter plots in case of certain models before weighting and after have been furnished in Appendix 4.1 to 4.2.

TABLE 5.8

VOLUME ESTIMATION MODELS (WEIGHTED)

SUMMARY OF REGRESSION STATISTICS (OVER BARK)

	SS	DF
Total	190.50	124

EQUATION	Regression SS	Regression DF	Root mean square error	SIGNIFICANCE OF COEFFICIENTS
5.1	109.13	5	.83	H, D, H^2, D^2
5.2	105.14	3	.84	H, D^2H
5.3	107.37	4	.83	D^2H
5.4	88.90	1	.91	D^2H *
5.5	110.13	4	.82	H^2, D^2H
5.6	105.16	2	.84	D^2H, D^3 *
5.7	105.17	3	.84	D^2H, D^3
5.9	107.37	5	.84	None
5.10	96.81	2	.88	D^2H, D^3H *
5.11	111.54	4	.81	H, H^2, D^2, D^2H *
5.14	90.74	2	.90	D^2H
5.15	106.26	3	.83	$D^2H, D^3H, (D^2H)^2$ *
5.16	105.18	3	.84	D^2H, D^3

* Indicate equations with all the coefficients significantly different from zero.

5.6 Grouping of Species

The present study provides an opportunity for an objective analysis of possible grouping of different tree species for the purpose of developing volume models. Thus, further analysis were carried out to see if there were differences between species in the volume equation itself. Species differences were incorporated in the volume equation through the use of dummy (0,1) variables and the interaction terms between these variables and the combined variable and other terms (as indicated in Table 5.1). However, out of 16 models only 4 viz, 5.4, 5.6, 5.10 and 5.11 were used for weighted covariance anlysis as only these had been proved to be satisfactory in the preceeding analysis of different models.

In the case of each model, these analyses involved fitting a regression which preserved the possibility of different intercepts for different species as well as different slopes for different species. This was termed the base model. At successive stages of the analysis, restrictions were introduced by the appropriate manipulation of dummy variables and various interactions to allow different progressive combinations of species groups. Thus restricted regressions were fitted with different slopes and different intercepts for each species combination (see Cunia, 1973). For example, Species Group 1 was combined with 2, Species Group 3 with 4 and the regression was fitted allowing different slopes and different intercepts for the two new groups. In the case of each model 12 possible variations in terms of different slopes, intercepts and species groups were analysed. Some details of these studies are shown in Appendix 1.1 to 1.4.

Regressions with all the coefficients significantly different from zero were selected for further testing. Null hypotheses were framed and the significance of different regressions was tested by the F-test. When no significant difference between two regressions could be established, the one with fewer terms was accepted. If two regressions were significantly different, the more precise one i.e. with lower mean square error (MSE) was chosen. The best forms of grouping for each model where regressions had coefficients significantly different from zero can be summarised thus:

Model No.	Regression	Grouping	Error	D.F.	MSE
5.4	2(iii)	1 and 2 vs 3 and 4	67.64	121	.559
5.6	4(iii)	1 and 2 vs 3 and 4	67.78	121	.560
5.11	4(iii)	1 and 2 vs 3 and 4	63.22	119	.531

In the case of all models 5.4, 5.6 and 5.11 the analysis lead to the same result with respect to grouping of species. All indicated that group 1 and 2 could be combined and likewise group 3 and 4. In other words, the rough-barked species could be grouped together but should be kept separate from smooth-barked species.Among the selected regressions from different models comparison had been carried out in similar manner.

Of the three models shown above model 5.11 seemed superior in that it had the lowest mean square error. Model 5.6 had higher error sum of squares compared to 5.4 for the same D.F. This was rejected. Test of the model 5.11 against 5.4 showed that this was significantly different from 5.4. Nevertheless, the model 5.11 was accepted because it had the lowest mean square error, in other words it was superior to either model 5.6 and 5.4.

Thus the final model for silver-top ash, messmate and yellow stringy-bark is:

$$V = .5705 \times 10^{-1} - .9735 \times 10^{-2}H + .1444 \times 10^{-3}D^2 + .3562 \times 10^{-3}H^2$$
$$(.2013 \times 10^{-1}) \quad (.2841 \times 10^{-2}) \quad (.6241 \times 10^{-4}) \quad (.9601 \times 10^{-4})$$
$$+ .1313 \times 10^{-4}D^2H$$
$$(.3226 \times 10^{-5})$$

that for southern blue gum and mountain grey gum is:

$$V = .6945 \times 10^{-1} - .9735 \times 10^{-2}H + .1444 \times 10^{-3}D^2 + .3562 \times 10^{-3}H^2$$
$$(.2067 \times 10^{-1}) \quad (.2841 \times 10^{-2}) \quad (.6241 \times 10^{-4}) \quad (.9601 \times 10^{-4})$$
$$+ .1313 \times 10^{-4}D^2H$$
$$(.3226 \times 10^{-5})$$

CHAPTER 6

UNDER BARK AND MERCHANTABLE HEIGHT MODELS

6.1 Models Based on Under Bark Diameter and Tree Height Measurements

6.1.1 General

Tree volume is expressed on an under bark basis in most inventories throughout the world. In the case of eucalypts, bark comprises a substantial part of the over bark volume. Tree volume models developed on the basis of under bark d.b.h. and height might therefore be expected to explain more variation in volume than those based on over bark data. The study data included both over bark and under bark measurements enabling further studies of model development on the basis of under bark measurements. A similar approach to that used for over bark models was followed. Because the basic principles and procedure are the same as in the previous chapter, they will not be dealt with in detail here.

6.1.2 Fitting of unweighted models

The volume models (1 to 16) listed in Table 5.1 were fitted to the combined data using d.b.h.u.b. in the place of d.b.h.o.b. Compared to over bark regressions these showed a general increase in the R^2 values as expected. Again, relevant tests indicated that only 5 models had all the coefficients significantly different from zero.

6.1.3 Weighting

As before, the homogeneity of error variance was tested using the Goldfeld-Quandt test and the presence of significant heteroscedasticity was established. Development of a separate weighting function was necessary for weighted least squares analysis in this case also. The results of volume variance and mean D^2H for each class are furnished in Table 6.1 and 6.2. Appropriate functions of an exponential form were developed to relate D^2H to variance of volume for three groups of species as before. As Table 6.3 shows, statistical analysis revealed no significant difference in the variance functions of the above groups. A variance function was developed combining data from different groups and is shown in Table 6.4, along with those for the individual species groups.

6.1.4 Fitting weighted models

Using the weighting function developed in the previous section, 13 models were fitted. A summary of regression statistics is shown in Table 6.5. The tables shows that only 5 models have all the coefficients significantly different from zero. Scatter plots of residuals against fitted values showed an almost uniform spread around zero. The Goldfeld-Quandt test was also applied and this was insignificant showing that the correction for heteroscedasticity had been effective.

TABLE 6.1

VARIANCE OF VOLUME AND MEAN OF D^2H CLASS

UNDER BARK					
Species Group 1		Species Group 2		Species Group 3&4 pooled	
$D^2H/1000$	Variance x 10000	$D^2H/1000$	Variance x 10000	$D^2H/1000$	Variance x 10000
.93	.45	.85	1.4	1.6	2.3
2.2	7.9	1.7	1.6	3.4	5.7
3.6	2.6	3.1	3.1	5.7	7.5
4.8	4.7	5.3	4.5	8.4	31.4
9.5	165.0	7.2	5.0	12.9	35.0
-	-	10.2	35.4		

TABLE 6.2

VARIANCE OF VOLUME

AND

MEAN OF D^2H CLASS

pooled and regrouped data
of Species Group 1,2,3 and 4

Under bark

$D^2H/1000$	Variance x 10000
.69	.25
1.06	.27
1.36	.14
1.79	.52
2.16	.21
2.96	.42
3.50	.83
3.94	.44
4.36	1.8
5.42	1.1
6.02	3.2
6.78	5.4
7.56	1.4
8.86	3.8
12.55	43.7

TABLE 6.3

COMPARISON OF VARIANCE FUNCTION
OF DIFFERENT SPECIES

[UNDER BARK]

Regression line	ESS	DF	MSE	F	Remarks
Between species 1 and species 2					
Species Group 1	.8581	3			Critical value of $F_{2,7}$ at .05P is 4.74
Species Group 2	.3365	4			
Pooled residuals	1.1946	7	.17066		
Species Group 1&2 (Single regression by pooling class means and variance of 1 and 2 together)	1.7050	9		1.495	Hence no significant difference exists.
Difference	.5104	2	.2552		
Between 1 and 2 combined and 3 and 4 combined					
Species Group 1 & 2	1.7050	9			Critical value of $F_{2,12}$ at .05 P is 3.89
Species Group 3 & 4	.0828	3			
Pooled residuals	1.7878	12	.1489		
Species Group 1,2,3 & 4 combined (Single regression by pooling class means and variance)	1.7990	14		.038	Hence no significant difference exists
Difference	.0112	2	.0056		

TABLE 6.4

VARIANCE ESTIMATION MODELS

(UNDER BARK)

SPECIES GROUP	MODEL
1	$\text{Log}_{10}\ (\text{Variance} \times 10000) = -.3116 + 2.130\ \text{Log}_{10}\ (D^2H/1000)$ (.4302) (.7067)
2	$\text{Log}_{10}\ (\text{Variance} \times 10000) = .06086 + 1.051\ \text{Log}_{10}\ (D^2H/1000)$ (.2061) (.3144)
3/4 (Pooled)	$\text{Log}_{10}\ (\text{Variance} \times 10000) = .04166 + 1.373\ \text{Log}_{10}\ (D^2H/1000)$ (.1815) (.2353)
Groups 1,2,3&4 with data pooled and regrouped	$\text{Log}_{10}\ (\text{Variance} \times 10000) = -.8329 + 1.566\ \text{Log}_{10}\ (D^2H/1000)$ (.1707) (.2653)

Standard error of estimates has been shown within brackets below in each case.

TABLE 6.5

VOLUME ESTIMATION MODELS (WEIGHTED)

SUMMARY OF REGRESSION STATISTICS (UNDER BARK)

	SS	DF
Total	545.70	124

EQUATION	Regression SS	DF	Root mean square error	SIGNIFICANCE OF COEFFICIENTS
5.1	477.24	5	.7585	H, D, D^2, DH
5.2	482.47	3	.7229	H, D^2H
5.3	487.16	4	.6984	D^2, D^2H
5.4	475.94	1	.7531	D^2H *
5.5	483.43	4	.7204	D^2H
5.6	483.89	2	.7117	D^2H, D^3 *
5.7	485.78	3	.7037	D^2H, D^3
5.9	487.16	5	.7000	D^2H
5.10	483.04	2	.7167	D^2H, D^3H *
5.11	484.92	4	.7117	D^2, H^2, D^2H
5.14	478.61	2	.7416	$D^2H, (D^2H)^2$ *
5.15	487.43	3	.6939	$D^2H, D^3H, (D^2H)^2$ *
5.16	484.60	3	.7106	D^2H, D^3

* Indicate all coefficients significant.

6.1.5 Grouping of species

Grouping of species for under bark volume models was studied using weighted covariance analysis with dummy variables defined in the same manner as in previous chapter. The analysis was carried out using the following weighted models.

Equation

5.4 $V = b_o + b_1D^2H$

5.6 $V = b_o + b_1D^2H + b_2D^3$

5.10 $V = b_o + b_1D^2H + b_2D^3H$

5.11 $V = b_o + b_1H + b_2D^2 + b_3H^2 + b_4D^2H$

5.14 $V = b_o + b_1D^2H + b_2(D^2H)^2$

5.15 $V = b_o + b_1D^2H + b_2D^3H + b_3(D^2H)^2$

Various possible combinations of species groupings, slope and level of regressions were analysed. The error sums of squares and corresponding D.F. are summarised in Appendix 2.1 to 2.6. The best forms of grouping for each of the following models can be summarised thus:

Model No.	Regression	Grouping	Error Sums of squares	D.F.	MSE
5.4	2(i)	1 and 2 combined, 3 and 4 separate.	61.59	119	.517
5.6	6(iii)	1 and 2 combined, 3 and 4 combined.	59.18	120	.493
5.10	4(iii)	1 and 2 combined, 3 and 4 combined.	62.13	121	.513
5.15	4(iii)	1 and 2 combined, 3 and 4 combined.	58.19	120	.485

None of the regressions from model 5.11 and 5.14 had coefficients significantly different from zero.

In the case of model 5.6, 5.10 and 5.15 the analysis led to the same result with respect to grouping of species, indicating that Groups 1 and 2 can be effectively combined and Groups 3 and 4 also can be combined together for the purpose of model development. Model 5.4 is the only one which had a different grouping. Among selected regressions from different models again comparison had been carried out with F-test.

Model 5.15 was used as a reference model, having the lowest value of mean square error. Model 5.6 was eliminated from consideration because model 5.15 which had the same degrees of freedom but lower mean square error was clearly superior to it. Model 5.10 was significantly different to model 5.15, but was rejected because it was markedly inferior in terms of mean square error. Model 5.15 was also markedly superior to the model 5.4 as with higher degrees of freedom it had lower error sum of squares compared to that of model 5.4. Thus, model 5.15 was chosen as it was the most precise.

Thus the final model chosen for silvertop ash, messmate and yellow stringy-bark is:

$$V = -.3103 \times 10^{-2} + .4446 \times 10^{-4}D^2H - .7193 \times 10^{-6}D^3H +$$
$$(.1193 \times 10^{-2}) \quad (.2387 \times 10^{-5}) \quad (.1818 \times 10^{-6})$$
$$.4749 \times 10^{-9}(D^2H)^2$$
$$(.1665 \times 10^{-9})$$

That for southern blue gum and mountain grey gum is:

$$V = -.2572 \times 10^{-2} + .4446 \times 10^{-4}D^2H - .7193 \times 10^{-6}D^3H +$$
$$(.1916 \times 10^{-2}) \quad (.2387 \times 10^{-5}) \quad (.1818 \times 10^{-6})$$
$$.4749 \times 10^{-9}(D^2H)^2$$
$$(.1665 \times 10^{-9})$$

6.2 Models Based on Diameter Over Bark and Merchantable Height

6.2.1 General

In tropical forests and hardwood forests of Australia tree height and merchantable volume under bark generally have only a loose correlation. If merchantable height can be measured accurately, volume models based on merchantable height can be expected to provide better estimates of merchantable volume.

6.2.2 Fitting unweighted models

Using merchantable height and d.b.h.o.b. as independent variables, 16 different models were fitted. Of these 4 models had all the coefficients significantly different from zero. From the study of scatter plot of residuals against fitted values it was found that in the case of all the models the error variance was not homogenous. The presence of heteroscedasticity was confirmed by Goldfeld-Quandt test.

6.2.3 Weighting

Detailed analyses were carried out to derive mean D^2H values for each class and corresponding values of volume variance and these are summarised in Table 6.6, while similar results for pooled data is furnished in Table 6.7. Exponential variance functions were fitted as before and analysis of covariance indicated that no significant difference exists between such functions from different Species Groups - see Table 6.8. Regression statistics, the value of the coefficients and estimated standard error for these functions are shown in Table 6.9.

6.2.4 Fitting weighted models

Weighted least squares analysis was performed using the above weighting function and 13 models as in previous cases. The resulting regression statistics are summarised in Table 6.10. Regression coefficients were found to be significantly different from zero in case of 6 models. Scatter plots and results of Goldfeld-Quandt test indicated that the error variance was homogenous.

TABLE 6.6

VARIANCE OF VOLUME AND MEAN OF D^2H CLASS

[MERCHATABLE HEIGHT AND DBH OVER BARK]

Species Group	$D^2H/1000$	variance x 10000
1	1.14	.54
	2.38	6.4
	4.52	1.1
	5.97	1.5
	8.16	5.6
	15.11	161.6
2	.98	1.1
	2.0	1.3
	3.5	4.2
	4.9	12.4
	7.9	8.9
	10.7	4.3
	16.1	34.8
3&4 Combined	1.8	2.2
	3.3	7.1
	5.3	5.9
	8.4	11.2
	13.8	59.8

6.2.5 Grouping of species

Analysis of covariance with dummy variables had been carried out using weighting function and the following models, some of the other promising models being omitted because of limitations on time:

Equation

5.4 $V = b_o + b_1D^2H$

5.2 $V = b_o + b_1H + b_2D^2 + b_3D^2H$

5.5 $V = b_o + b_1H + b_2D^2H + b_3H^2 + b_4D^2H^2$

5.10 $V = b_o + b_1D^2H + b_2D^3H$

Various grouping possibilities and forms of regression with respect to slope and level were again studied. Results are summarised in Appendix 3.1 to 3.4. Various regressions were compared with F-test. The following regressions for each model were selected:

Model No.	Regression	Grouping	Error sum of squares	D.F.	MSE
5.4	6(iii)	1 and 2 vs 3 and 4	87.27	122	.715
5.5	4(iii)	1 and 2 vs 3 and 4	81.20	119	.682
5.10	4(iii)	1 and 2 vs 3 and 4	89.19	121	.737

Model 5.5 was used as a reference model being the one with lowest mean square error. Model 5.4 proved to be significantly different from 5.5, but the former being less precise had been rejected.

TABLE 6.7

VARIANCE OF VOLUME AND MEAN OF D^2H CLASS

[MERCHANTABLE HEIGHT AND DBH OVER BARK]

	$D^2H/100$	variance x 10000
1	.78	.45
2	1.24	.91
3	1.69	.51
4	2.26	1.6
5	2.83	3.1
6	3.82	2.2
7	4.35	6.4
8	4.75	2.3
9	5.45	7.5
10	6.80	5.3
11	7.83	1.3
12	8.49	6.5
13	9.54	7.3
14	11.58	17.0
15	16.93	56.0

Combined and regrouped data of Sp. Group1,2,3&4

TABLE 6.8

COMPARISON OF VARIANCE FUNCTION OF DIFFERENT SPECIES

[MERCHANTABLE HEIGHT AND DBH OVER BARK]

Between species 1 and species 2

Regression line	ESS	DF	MSE	F	Remarks
Species Group 1	1.794	4			
Species Group 2	.4558	5			Critical value of $F_{2,9}$ at .05 P is 4.26
Pooled residuals	2.2498	9	.2500		
Species Group 1&2 (Single regression by pooling class means and variance of 1&2 together)	2.419	11		.338	Hence no significant difference exists.
Difference	.1692	2	.0846		

Between 1 and 2 combined and 3 and 4 combined

Regression line	ESS	DF	MSE	F	Remarks
Species Group 1&2	2.4190	11			
Species Group 3&4	.1546	3			
Pooled residuals	2.5736	14	.1838		Critical value of $F_{2,14}$ at .05 P is 3.74
Group 1,2,3&4 combined (Single regression by pooling class means and variance)	2.7600	16		.507	Hence no significant difference exists.
Difference	.1864	2	.0932		

TABLE 6.9

VARIANCE ESTIMATION MODELS

[MERCHATABLE HEIGHT AND DBH OVER BARK]

Summary of regression statistics

Equation	Degrees of freedom		Sums of squares	
	Total	Regression	Total	Regression
Species Group 1	5	1	3.903	2.209
Species Group 2	6	1	1.714	1.2582
Species Group 3 & 4 combined	4	1	1.113	.9584
Species Group 1,2,3 & 4 combined and data regrouped	14	1	4.526	3.371

Group	Models
1	Log_{10} (Variance x 10000) = $\underset{(.5698)}{-.4641} + \underset{(.7555)}{1.638}\ \mathrm{Log}_{10}$ $(D^2H/1000)$
2	Log_{10} (Variance x 10000) = $\underset{(.2237)}{-.3713} \times 10^{-2} + \underset{(.2891)}{1.074}\ \log_{10}$ $(D^2H/1000)$
3 & 4	Log_{10} (Variance x 10000) = $\underset{(.2572)}{-.6128} \times 10^{-1} + \underset{(.3320)}{1.432}\ \log_{10}$ $(D^2H/1000)$
1,2,3 & 4 combined	Log_{10} (Variance x 10000) = $\underset{(.1552)}{-.3048} + \underset{(.2112)}{1.301}\ \mathrm{Log}_{10}$ $(D^2H/1000)$

TABLE 6.10

VOLUME ESTIMATION MODELS (WEIGHTED)

SUMMARY OF REGRESSION STATISTICS [MERCHANTABLE HEIGHT, DBH(OB)]

	SS	DF
Total	430.30	124

EQUATION	Regression SS	Regression DF	Root mean square error	SIGNIFICANCE OF COEFFICIENTS
5.1	342.29	5	.8599	H, D, D^2
5.2	323.50	3	.9394	H, D^2, D^2H *
5.3	324.60	4	.9385	D^2H
5.4	312.80	1	.9773	D^2H *
5.5	328.70	4	.9201	H, D^2H, H^2, D^2H^2 *
5.6	313.00	2	.9805	D^2H
5.7	323.40	3	.9399	D^2H, D^3, D^2 *
5.9	324.90	5	.9411	None
5.10	323.20	2	.9369	D^2H, D^3H *
5.11	339.51	4	.8698	H, H^2, D^2
5.14	321.80	2	.9430	$D^2H, (D^2H)^2$ *
5.15	323.20	3	.9408	$D^2H, (D^2H)^2$
5.16	321.90	3	.9465	$D^2H, (D^2H)^2$

* These models have all the coefficients significantly different from zero.

Similarly model 5.10 was significantly different from model 5.5 but was less precise than 5.5. Hence model 5.5 was chosen as the best model. Thus the selected model for silver-top ash, messmate and yellow stringy-bark is:

$$
\begin{aligned}
V = {} & .5106 \times 10^{-1} - .1054 \times 10^{-1} H + .3761 \times 10^{-4} D^2 H + \\
& (.1398 \times 10^{-1}) \quad (.2807 \times 10^{-2}) \quad (.3001 \times 10^{-5}) \\
& .4848 \times 10^{-3} H^2 - .7474 \times 10^{-6} D^2 H^2 \\
& (.1290 \times 10^{-3}) \quad (.1809 \times 10^{-6})
\end{aligned}
$$

That for southern blue gum and mountain grey gum is:

$$
\begin{aligned}
V = {} & .6705 \times 10^{-1} - .1054 \times 10^{-1} H + .3761 \times 10^{-4} D^2 H + \\
& (.1461 \times 10^{-1}) \quad (.2807 \times 10^{-2}) \quad (.3001 \times 10^{-5}) \\
& .4848 \times 10^{-3} H^2 - .7474 \times 10^{-6} D^2 H^2 \\
& (.1290 \times 10^{-3}) \quad (.1809 \times 10^{-6})
\end{aligned}
$$

CHAPTER 7

CONCLUSIONS

As indicated in the previous chapters, 3 different models showed the best fit for the data in the case of diameter over bark, diameter under bark and merchantable height measurements. It is found that the form of model suitable in the above three cases is quite different. The grouping of species in each model is the same, in other words, grouping of two rough-bark species and grouping of two smooth-bark species have been found to be appropriate.

The difference in the form of the model may reflect the difference in variables concerned.

Furthermore, further testing of these equations would be desirable, based on independent data. In the absence of any such data the accuracy of the models could not be fully tested. Nevertheless, the models all have coefficients of sensible sign and magnitude and explained a very high proportion of the variance in volume. Thus, they are expected to perform reasonably well.

The model developed on the basis of diameter under bark was more precise than those based on diameter over bark. The former explained more variance in volume as expected. However, use of diameter under bark models require accurate measurements of bark thickness on standing trees, which in practice is extremely difficult. During this project, the author studied this aspect in the forests concerned. Accurate measurement of bark thickness of eucalypt trees with two different Swedish bark gauges was practically impossible. Better instruments would have to be developed if diameter under bark is to be used in field inventories in eucalypt forests. Given a suitable device for accurate measurements of bark thickness, under bark models will be more efficient in volume estimation.

The model developed on the merchantable height is more precise than that based on total tree height. As merchantable volume involved varying diameter limits in this study, this result was anticipated. Total height measurement in the case of trees of deliquescent form is difficult due to the lack of a distinct leading shoot. On the otherhand, assessment of merchantable height in standing trees is generally imprecise due to the difficulty of identifying the merchantable limit in case of tropical hardwoods and eucalypts. Where merchantable height can be specifically defined and identified in the field, it should be used in volume prediction models.

In the perspective of Indian Forests, certain aspects of the present study need mention. The concept of weighted multiple regression analysis for tree volume estimation is comparatively recent, though the need for such an approach has long been felt. Developing the most appropriate weighting function in the case of commercial tree species of India may need more elaborate study. The exponential error variance function used in this study may not be the best for other tropical hardwoods. In the case of forest inventories in India dealing with numerous tree species, some preliminary grouping of species on the basis of scatter plot of tree parameters seems advisable. Final grouping studies with a limited number of

Species Groups may then be carried out using the same analysis as used in this study.

REFERENCES

Aldrich, R.C. (1966). Forestry applications of 70mm colour photography. Photogramm. Engng. 37: 389-401.

Archer, G.R. (1977). Tree volume models for tropical broad-leaved forests. M.Sc thesis, A.N.U., unpublished.

Bamping, J.H. (1974). The manager's role. Proc. Soc. Am. For., Inventory Working Group, meeting Colo. State Univ. July 23-25. 1974: 4-9.

Banks, P.F. and Burrows, P.M. (1966). Preliminary local volume table for *Baikiaea plurijuga, Guibourtia coleosperma* and *Pterocarpus angolensis* in Rhodesia. Comm. For. Rev. 45(3): 256-264.

Barnard, J.E., Bickford, C.A. and Mayer, C.E. (1973). Forest survey cubic-foot volume equations. USDA Forest Service Research Note NE-166.

Bartlett, M.S. (1937). Properties of sufficiency and statistical tests. Proc. Roy. Soc. Lond. Series A. 160: 268-282.

Bickford, C.A. (1966). Sampling methods in relation to forest inventory. Proc. 15th Annual For. Symp., Louisiana St. Univ., 1966: 121-135.

Bonnor, G.M. (1964). A tree volume table for Red Pine by crown width and height. For. Chron. 40(3): 339-346.

Bowling, P.J. (1951). Volume tables for regrowth of some Tasmanian eucalypts. Aust. For. 15(2): 87-95.

Brown, J.E. (1962). Standard cubic foot volumle tables for the commercial tree species of British Columbia. Forest surveys and Inventory Division, B.C. Forest Service.

Bruce, D. and Schumacher, F.X. (1950). Forest mensuration. McGraw-Hill Book Co. Inc., New York: 484pp.

Brunig, E.W.F. (1963). The history of forest invnetories in Sarawak. Mal. For. 26: 141-158.

Burley, J., Wright, H.L. and Matos, E. (1972). Volume table for *Pinus caribaea* var *caribaea*. Comm. For. Rev. 51(2): 137-143.

Carron, L.T. (1968). An outline of forest mensuration with special reference to Australia. ANU Press, Canberra: 224pp.

Chaturvedi, A.N. (1973). General standard volume tables and height-diameter relationships for teak (*Tectona grandis*). Indian Forest Records 12(8).

Cochrane, D. and Orcutt, G.H. (1949). Application of least-squares regression to relationships containing autocorrelated error terms. J. Amer. Statist. Assoc. 44: 32-61.

Cochran, W.G. (1941). The distribution of the largest of a set of estimated variances as a function of their total. Ann. Eugenics 11: 45-52.

Cochran, W.G. (1953). Sampling techniques. John Wiley & Sons Inc., New York: 330pp.

Cromer, D.A.N., McIntyre, G.A. and Lewis, N. (1955). General volume table for *Pinus radiata*. Forestry and Timber Bureau Bulletin No. 33.

Cunia, T. (1964). Weighted least squares method and construction of volume tables. For. Sci. 10(2): 180-191.

Cunia, T. (1973). Dummy variables and some of their uses in regression analysis. Proceedings of the I.U.F.R.O. subject group S4.02, Nancy, France, June 25-29, 1973, Volume 1.

Dargavel, J.B. (1969). Inventory and yield forecasting for industrial pine plantation in Gippsland, Victoria. M.Sc (For) thesis, Univ. Melbourne, 1969, unpublished.

Dargavel, J.B. and Ditchburne, N. (1971). A comparison of the volumes of tree stems obtained by direct measurement and by water displacement. Aust. For. 35(3): 191-198.

Dixon, W.J. and Massey, F.J. (1957). Introduction to statistical analysis. 2nd ed. McGraw-Hill, New York.

Dowden, H. (1967). Estimating internal defect growth in living trees. Aust. For. Res. 4(3): 11-13.

Draper, N.R. and Smith, H (1966). Applied regression analysis. Wiley, New York: 407pp.

Durbin, J. and Watson, G.S. (1950). Testing for serial correlation in least squares regression. I. Biometrica 37: 409-428.

Durbin, J. and Watson, G.S. (1951). Testing for serial correlation in least squares regression. II. Biometrica 38: 159-178.

Ezekial, M. and Fox, K.A. (1959). Methods of correlation and regression analysis. Wiley, New York.

FAO (1973). Mannual of Forest inventory, with special reference to mixed tropical forests. FAO, Rome: 200pp.

Freese, F. (1964). Linear regression mehtods for forest research. U.S. Forest Service Research Paper. FPL 17: 136pp.

Freese, F. (1967). Elementary statistical mehtods for foresters. U.S. Forest Service Agric. Handbook 317: 87pp.

Furnival, G.M. (1961). An index for comparing equations used in constructing volume tables. For. Sci. 7(4): 337-341.

Gedney, D.R. and Johnson, F.A. (1959). Weighting factors for computing the relation between tree volume and d.b.h. in the Pacific Northwest. Pac. N. W. Forest and Range Expt. Sta: 5pp.

Gerrard, D.J. (1966). The construction of standard tree volume tables by weighted multiple regression. Uni. Toronto Tech. Rept. 6: 35pp.

Gevorkiantz, S.R. and Olsen, L.P. (1955). Composite volume tables for timber and their application to the Lake States. For. Service, U.S.D.A. Tech. Bull. 1104: 51pp.

Gibson, B.F. and Webb, A.W. (1968). Regression methods and models for estimating tree volumes of Alpine Ash regrowth. Aust. For. 32(4): 233-241.

Glejser, H. (1969). A new test for heteroscedasticity. J. Amer. Statist. Assoc., Vol 64, 1969: 316-323.

Goldfeld, M.S. and Quandt, R.E. (1965). Some tests for homoscedasticity. J. Amer. Statist. Assoc., June, 1965: 539-547.

Golding, D.L. and Hall, O.F. (1961). Tests of precision of cubic foot tree volume equations on aspen, Jack pine, and white spruce. For. Chron. 37: 123-132.

Government of India (1976). Indian Agriculture in brief. 1976. Directorate of Economics and Statistics, Ministry of Agriculture and Irrigation, New Delhi.

G.O.I. (1979). Pre-investment Survey of Forest Resources. Government of India, Ministry of Agriculture and Irrigation, 1979. Dehra Dun: 5pp.

Grosenbaugh, L.R. (1963). Optical dentrometers for out of reach diameters: a conspectus and some new theory. For. Sci. Monograph. No. 4.

Hamilton, G.J. (1975). Forest mensuration handbook. For. Commission booklet No. 39. London, 1975: 274pp.

Hartley, H.O. (1950). Maximum F-ratio as a shortcut test for heterogeneity of variance. Biometrika 37, p 308-312.

Hegyi, F. (1965). Taper index class composite volume tables. Comm. For. Rev. 44(3): 193-197.

Henry, J.L. (1960). Volume table preparation for *Pinus radiata* in N.S.W. Paper presented to First Aust. Comp. Soc. Conf. Sydney.

Henshaw, R.C. (Jr.) (1966). Testing single equation least squares regression models for autocorrelated distrubances. Econometrica, Vol. 34, 1966: 646-660.

Hindley, E.W. (1973). The evaluation of a boring-for-decay program undertaken in a Malaysian tropical forest. Proceedings of the I.U.F.R.O. subject group S4.02, Nancy, France, June 25-29, 1973. Vol. 3: 1-49.

Honer, T.G. (1965). A new total cubic foot volume function. For. Chron. 41: 476-493.

Howard, J.A. and Lanly J.P. (1975). Remote sensing in tropical forest surveys. Unasylva, 27(108): 32-37.

Husch, B. (1963). Forest mensuration and statistics. Ronald Press, New York: 474pp.

Husch, B., Miller, C.I. and Beers, T.W. (1972). Forest mensuration 2nd ed. Ronald Press, New York: 410pp.

Ilvessalo, Y. (1947). Volume tables for standing trees. Comm. Inst. For. Fenn., 34(4): 1-149.

Johnston, J. (1972). Econometric methods. International student edition. McGraw-Hill-Kogakusha Ltd, Tokyo: 437pp.

King, K.F.S. (1975). Putting the emphasis on tropical forestry. Unasylva 27(110): 30-35.

Leech, J.W. (1973). Tree volume and increment models for radiata pine thinnings. M.Sc thesis, A.N.U., unpublished.

Lyons, E.H. (1966). Fixed airbase 70mm photography, a new tool for forest sampling. For. Chron. 4, 1966: 420-431.

Meyer, H.A. (1953). Forest mensuration. Penns. Valley Publishers Inc. State College, Pennsylvania.

Miller, C.I. (1959). Comparison of Newton's, Smalian's and Huber's formulas. Dept. of Forestry and Conservation, Purdue University, U.S.A.

Moser, J.W. (Jr.) and Beers, T.W. (1969). Parameter estimation in nonlinear volume equations. J. For. 67(12): 879.

Munro, D.A. (1964). Weighted least squares solutions imporve precision of tree volume estimates. For. Chron. 40: 400-401.

Nash, A.J. (1965). Statistical techniques in Forestry. Lucas Brothers. Columbia, Missouri: 146pp.

Nash, A.J. (1973). Inventory methods in tropical rainforests of Surinam. Proceedings of the I.U.F.R.O. subject group S4.02, Nancy, France, June 25-29, 1973. Volume 3.

Neter, J. and Wasserman, W. (1974). Applied linear statistical models. Richard D. Irwin, Inc. Illinois: 842pp.

Nilsson, N.E. (1973). Forest inventory - examples on the formulation of detailed objectives. Lecture notes, FAO/SIDA Training course on Forest Inventory: 1973.

Odum, E.P. (1971). Fundamentals of ecology. International student edition, Toppan Co. Ltd, Tokyo.

Pindyck, R.S. and Rubinfeld, D.L. (1976). Econometric models and economic forecasts. McGraw-Hill, New York: 576pp.

P.I.S.F.R. (1976). Report on the forest resources of Kameng and Subansiri districts of Arunachal pradesh. Preinvestment Survey of Forest Resources, Government of India, Dehra Dun.

Sandrasegaran, K. (1969). A general volume table for *Tectona grandis* Linn (f) (teak) grown in north-west Malaya. Mal. For. 32: 187-200.

Sayn-Wittgenstein, L. and Aldred, A.H. (1967). Tree volumes from large scale photos. Photogramm. Engng. 33(1): 69-73.

Schimitt, D. and Bower, D. (1970). Volume tables for young loblolly, slash and long leaf pines in plantations in south Missisipi. U.S.D.A Forest Service Research Note. South For. Exp. Station.

Schumacher, F.X. and Hall, F. (1933). Logarithmic expression of timber tree volume. Jour. Agr. Res. 47: 719-734.

Shapiro, S.S. and Wilk, M.B. (1965). An analysis of variance test for normality (complete samples). Biometrika 52: 591-611.

Shapiro, S.S., Wilk, M.B. and Chen, Mrs. H.J. (1968). A comprehensive study of various tests for normality. J. Amer. Statist. Assoc. 63: 1343-1372.

Sharma, R.P. and Jain, R.C. (1978). Regional volume tables of four native tree species of north-eastern region of India. Indian Forester, 1978(11): 734-743.

Sharma, R.P. and Jain, R.C. (1979). Regional volume tables of six native tree species of north-eastern region, India. Indian Forester, 1979(3): 203-210.

Shield, E.D. (1965). The application of new sampling methods to previously inaccessible tropical forest areas with particular reference to Papua New Guinea. Thesis for Dip. For. Oxford, unpublished.

Silva, J.A. DA. (1976). Relation between diameter and height and evaluation of random errors in volume of a stand of *Auraucaria angustifolia*. Revista do centro de ciencias. Rurais (1976) 6(3). Univ. Federal de Santa Maria, Brazil.

Smalley, G.W. (1973). Weighting tree volume equations for young loblolly and short leaf pines. U.S.D.A. Forest Service Research Note, South. For. Exp. Station.

Smith, J.H.G., Ker, J.W. and Csizmazia, J. (1961). Economics of reforestation of douglas fir, western hemlock and western red cedar in the Vancouver forest district. Forestry Bulletin No. 3, Univ. of British Columbia, Canada.

Snedecor, G.W. and Cochran, W.G. (1967). Statistical mehtods. 6th ed. Iowa State University Press, Ames, Iowa, U.S.A.

Sokal, R.R. and Rohlf, F.J. (1969). Biometry. W.H. Freeman, San Francisco: 776pp.

Spurr, S.H. (1952). Forest Inventory. Ronald press, New York: 476pp.

Stage, A.R., Dodge, R.C. and Brickell, J.E. (1968). NETVSL - a computer programme for calculation of tree volumes with interior defect. U.S.D.A. Forest Service Research Paper INT-51.

Stoate, T.N. (1945). The use of a volume equation in pine stands. Aust. For. 9(1): 48-52

Sukhatme, P.V. (1954). Sampling theory of surveys with applications. Iowa State College Press Ames.

Theil, H. and Nagar, A.L. (1961). Testing the independence of regression disturbances. J. Amer. Statist. Assoc. 56: 793-806.

Theil, H. (1971). Principles of econometrics. John Wiley and Sons, New York.

Thornber, W. (1948). Tree volume determination. Pulp and Pap. Res. Inst. Canada, Woodlands Res. Index 41(B-6): 2pp.

Turner, B.J. (1972). Board-foot and cubic-foot volume tables for the commercial forest species of Pennsylvania. College of Agriculture, Pennsylvania State University.

Unwin, P.T. and Bowling, P.J. (1951). Production of volume tables for mature trees in Tasmania, Aust. For. 15(1): 31-41.

Van Schie, R. (1967). Measuring standing sample trees with a dendrometer in regrowth forests of southern Tasmaia. Aust. For., 31(3): 181-192.

Wong, Y.K. (1966). Possibility of using a multi-species local volume table to estimate the total volumes in Lowland Dipterocarp forest. Mal. For. 29(4): 259-267.

Wonnacott, R.J. and Wonnacott, T.H. (1970). Econometrics. John Wiley, New York.

Wood, G.B. (1980). Lecture notes on principles of forest measurement. A.N.U. Canberra.

Young, H.E., Robbins, W.C. and Wilson, S. (1967). Errors in volume determination of primary forest products. I.U.F.R.O. Proc. 14th Congress, Munich, Part VI, Section 25: 546-562.

REGRESSION STATISTICS OF WEIGHTED COVARIANCE ANALYSIS USING DUMMY VARIABLES (OVER BARK)

EQUATION: $V = b_0 + b_1 D^2 H$

TSS 190.5 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopesand different intercepts.	64.09	117
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	64.15	119
ii) 3 and 4 combined, 1 and 2 separate.	67.58	119
iii) 1 and 2 combined, 3 and 4 combined.	67.64	121
3. Maximum model with same slope and different intercepts.	70.36	120
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	70.39	121
ii) 3 and 4 combined, 1 and 2 separate.	71.96	121
iii) 1 and 2 combined, 3 and 4 combined.	71.99	122
5. Maximum model with same intercept and different slopes.	68.33	120
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	68.34	121
ii) 3 and 4 combined, 1 and 2 separate.	70.90	121
iii) 1 and 2 combined, 3 and 4 combined.	70.90	122

1, 2, 3, 4 indicate respective Species Group:

All the regressions have coefficient significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED COVARIANCE ANALYSIS USING DUMMY VARIABLES (OVER BARK)

EQUATION: $V = b_0 + b_1D^2H + b_2D^3$

TSS 190.5 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	58.60	113
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	58.71	116
ii) 3 and 4 combined, 1 and 2 separate.	62.65	116
iii) 1 and 2 combined, 3 and 4 combined.	62.76	119
3. Maximum model with same slope and different intercepts.	67.07	119*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	67.32	120*
ii) 3 and 4 combined, 1 and 2 separate.	67.49	120*
iii) 1 and 2 combined, 3 and 4 combined.	67.78	121*
5. Maximum model with same intercept and different slopes.	62.42	116
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	62.42	118
ii) 3 and 4 combined, 1 and 2 separate.	62.95	118
iii) 1 and 2 combined, 3 and 4 combined.	62.96	120

1, 2, 3, 4 indicate respective Species Group:

* Indicate regressions with coefficient significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

(OVER BARK)

EQUATION: $V = b_0 + b_1D^2H + b_2D^3H$

TSS 190.5 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	59.60	113
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	60.64	116
ii) 3 and 4 combined, 1 and 2 separate.	63.29	116
iii) 1 and 2 combined, 3 and 4 combined.	64.32	119
3. Maximum model with same slope and different intercepts.	68.50	119
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	68.62	120
ii) 3 and 4 combined, 1 and 2 separate.	69.94	120
iii) 1 and 2 combined, 3 and 4 combined.	70.08	121
5. Maximum model with same intercept and different slopes.	61.28	116
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	61.89	118
ii) 3 and 4 combined, 1 and 2 separate.	63.71	118
iii) 1 and 2 combined, 3 and 4 combined.	64.36	120

1, 2, 3, 4 indicate respective Species Group:

None of the regressions above had all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

(OVER BARK)

EQUATION: $V = b_0 + b_1H + b_2D^2 + b_3H^2 + b_4D^2H$

TSS 190.5 DF 124

REGRESSION	SS	DF
1. Maximum model with different slopes and different intercepts	54.79	105
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	56.25	110
ii) 3 and 4 combined, 1 and 2 separate.	58.98	110
iii) 1 and 2 combined, 3 and 4 combined.	60.44	115
3. Maximum model with same slope and different intercepts.	62.85	117*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	63.11	118*
ii) 3 and 4 combined, 1 and 2 separate.	63.02	118*
iii) 1 and 2 combined, 3 and 4 combined.	63.22	119*
5. Maximum model with same intercept and different slopes.	56.36	108
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	57.97	112
ii) 3 and 4 combined, 1 and 2 separate.	59.80	112
iii) 1 and 2 combined, 3 and 4 combined.	61.67	116

1, 2, 3, 4 indicate respective Species Group:

* Indicate regressions with coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES (UNDER BARK)

EQUATION: $V = b_0 + b_1 D^2 H$

TSS 545.7 DF 124

REGRESSION	E SS	DF
1. Maximum model with different slopes and different intercepts.	59.43	117
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	61.59	119
ii) 3 and 4 combined, 1 and 2 separate.	62.98	119
iii) 1 and 2 combined, 3 and 4 combined.	65.15	121
3. Maximum model with same slope and different intercepts.	66.60	120
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	67.60	121
ii) 3 and 4 combined, 1 and 2 separate.	66.84	121
iii) 1 and 2 combined, 3 and 4 combined.	67.84	122
5. Maximum model with same intercept and different slopes.	66.14	120
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	68.26	121
ii) 3 and 4 combined, 1 and 2 separate.	67.64	121
iii) 1 and 2 combined, 3 and 4 combined.	69.76	122

1, 2, 3, 4 indicate respective Species Group:

All the above regressions have coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES
(UNDER BARK)

EQUATION: $V = b_0 + b_1D^2H + b_1D^3$

TSS 545.7 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	50.27	113
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	52.90	116
ii) 3 and 4 combined, 1 and 2 separate.	54.82	116
iii) 1 and 2 combined, 3 and 4 combined.	57.46	119*
3. Maximum model with same slope and different intercepts.	61.06	119*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	61.43	120*
ii) 3 and 4 combined, 1 and 2 separate.	61.12	120*
iii) 1 and 2 combined, 3 and 4 combined.	61.50	121*
5. Maximum model with same intercept and different slopes.	56.26	116
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	58.84	118*
ii) 3 and 4 combined, 1 and 2 separate.	56.61	118
iii) 1 and 2 combined, 3 and 4 combined.	59.18	120*

1, 2, 3, 4 indicate respective Species Group:

* Indicate regressions with all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

(UNDER BARK)

EQUATION: $V = b_0 + b_1D^2H + b_2D^3H$

TSS 545.7 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	52.86	113
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	55.59	116
ii) 3 and 4 combined, 1 and 2 separate.	58.12	116
iii) 1 and 2 combined, 3 and 4 combined.	60.85	119
3. Maximum model with same slope and different intercepts.	61.52	119*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	61.98	120*
ii) 3 and 4 combined, 1 and 2 separate.	61.67	120*
iii) 1 and 2 combined, 3 and 4 combined.	62.13	121*
5. Maximum model with same intercept and different slopes.	57.01	116
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	59.48	118*
ii) 3 and 4 combined, 1 and 2 separate.	58.81	118
iii) 1 and 2 combined, 3 and 4 combined.	61.29	120*

1, 2, 3, 4 indicate respective Species Group:

* Indicate regressions with all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

(UNDER BARK)

EQUATION: $V = b_0 + b_1H + b_2D^2 + b_3H^2 + b_4D^2H$

TSS 545.7 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	45.41	105
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	50.73	110
ii) 3 and 4 combined, 1 and 2 separate.	49.62	110
iii) 1 and 2 combined, 3 and 4 combined.	54.93	115
3. Maximum model with same slope and different intercepts.	60.16	117
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	60.58	118
ii) 3 and 4 combined, 1 and 2 separate.	60.28	118
iii) 1 and 2 combined, 3 and 4 combined.	60.74	119
5. Maximum model with same intercept and different slopes.	48.66	108
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	53.83	112
ii) 3 and 4 combined, 1 and 2 separate.	50.86	112
iii) 1 and 2 combined, 3 and 4 combined.	56.87	116

1, 2, 3, 4 indicate respective Species Group:

None of the above regressions had all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES
(UNDER BARK)

EQUATION: $V = b_0 + b_1 D^2H + b_2 (D^2H)^2$

TSS 545.7 DF 124

	REGRESSION	ESS	DF
1.	Maximum model with different slopes and different intercepts.	55.98	113
2.	Restricted model with different slopes and different intercepts.		
	i) 1 and 2 combined, 3 and 4 separate.	58.56	116
	ii) 3 and 4 combined, 1 and 2 separate.	60.51	116
	iii) 1 and 2 combined, 3 and 4 combined.	63.09	119
3.	Maximum model with same slope and different intercepts.	64.69	119
4.	Restricted model with same slope and different intercepts.		
	i) 1 and 2 combined, 3 and 4 separate.	65.44	120
	ii) 3 and 4 combined, 1 and 2 separate.	65.02	120
	iii) 1 and 2 combined, 3 and 4 combined.	65.78	121
5.	Maximum model with same intercept and different slopes.	60.96	116
6.	Restricted model with same intercept and different slopes.		
	i) 1 and 2 combined, 3 and 4 separate.	63.36	118
	ii) 3 and 4 combined, 1 and 2 separate.	64.38	118
	iii) 1 and 2 combined, 3 and 4 combined.	66.76	120

1, 2, 3, 4 indicate respective Species Group:

None of the above regressions had all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

(UNDER BARK)

EQUATION: $V = b_0 + b_1D^2H + b_2D^3H + b_3(D^2H)^2$

TSS 545.7 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts	49.14	109
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	51.66	113
ii) 3 and 4 combined, 1 and 2 separate.	52.05	113
iii) 1 and 2 combined, 3 and 4 combined.	54.57	117
3. Maximum model with same slope and different intercepts.	57.93	118*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	58.17	119*
ii) 3 and 4 combined, 1 and 2 separate.	57.94	119*
iii) 1 and 2 combined, 3 and 4 combined.	58.19	120*
5. Maximum model with same intercept and different slopes.	51.72	112
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	54.17	115
ii) 3 and 4 combined, 1 and 2 separate.	52.34	115
iii) 1 and 2 combined, 3 and 4 combined.	54.79	118

1, 2, 3, 4 indicate respective Species Group:

*Indicates regressions with coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

[MERCHANTABLE HEIGHT AND DBH(OB)]

EQUATION: $V = b_0 + b_1D^2H$

TSS 430.3 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	83.16	117
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	83.42	119
ii) 3 and 4 combined, 1 and 2 separate.	85.35	119
iii) 1 and 2 combined, 3 and 4 combined.	85.61	121
3. Maximum model with same slope and different intercepts.	91.40	120
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	91.73	121
ii) 3 and 4 combined, 1 and 2 separate.	92.90	121
iii) 1 and 2 combined, 3 and 4 combined.	93.23	122
5. Maximum model with same intercept and different slopes.	85.45	120
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	85.64	121
ii) 3 and 4 combined, 1 and 2 separate.	87.08	121
iii) 1 and 2 combined, 3 and 4 combined.	87.27	122

1, 2, 3, 4 indicate respective Species Group:

All be above regressions have coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

[MERCHANTABLE HEIGHT AND DBH(OB)]

EQUATION: $V = b_0 + b_1H + b_2D^2 + b_3D^2H$

TSS 430.3 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	54.06	109
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	57.72	113
ii) 3 and 4 combined, 1 and 2 separate.	59.71	113
iii) 1 and 2 combined, 3 and 4 combined.	63.37	117
3. Maximum model with same slope and different intercepts.	79.81	118
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	79.85	119
ii) 3 and 4 combined, 1 and 2 separate.	84.54	119
iii) 1 and 2 combined, 3 and 4 combined.	84.71	120
5. Maximum model with same intercept and different slopes.	55.80	112
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	58.71	115
ii) 3 and 4 combined, 1 and 2 separate.	60.57	115
iii) 1 and 2 combined, 3 and 4 combined.	63.48	118

1, 2, 3, 4 indicate respective Species Group:

None of the above regressions had all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES
[MERCHANTABLE HEIGHT AND DBH(OB)]

EQUATION: $V = b_0 + b_1H + b_2D^2H + b_3H^2 + b_4D^2H^2$

TSS 430.3 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	53.49	105
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	59.16	110
ii) 3 and 4 combined, 1 and 2 separate.	58.65	110
iii) 1 and 2 combined, 3 and 4 combined.	64.31	115
3. Maximum model with same slope and different intercepts.	79.66	117*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	79.76	118*
ii) 3 and 4 combined, 1 and 2 separate.	81.12	118*
iii) 1 and 2 combined, 3 and 4 combined.	81.20	119*
5. Maximum model with same intercept and different slopes.	55.60	108
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	61.11	112
ii) 3 and 4 combined, 1 and 2 separate.	61.03	112
iii) 1 and 2 combined, 3 and 4 combined.	66.66	116

1, 2, 3, 4 indicate respective Species Group:

* Indicates regressions with all the coefficients significantly different from zero.

REGRESSION STATISTICS OF WEIGHTED

COVARIANCE ANALYSIS USING DUMMY VARIABLES

[MERCHANTABLE HEIGHT AND DBH(OB)]

EQUATION: $V = b_0 + b_1D^2H + b_2D^3H$

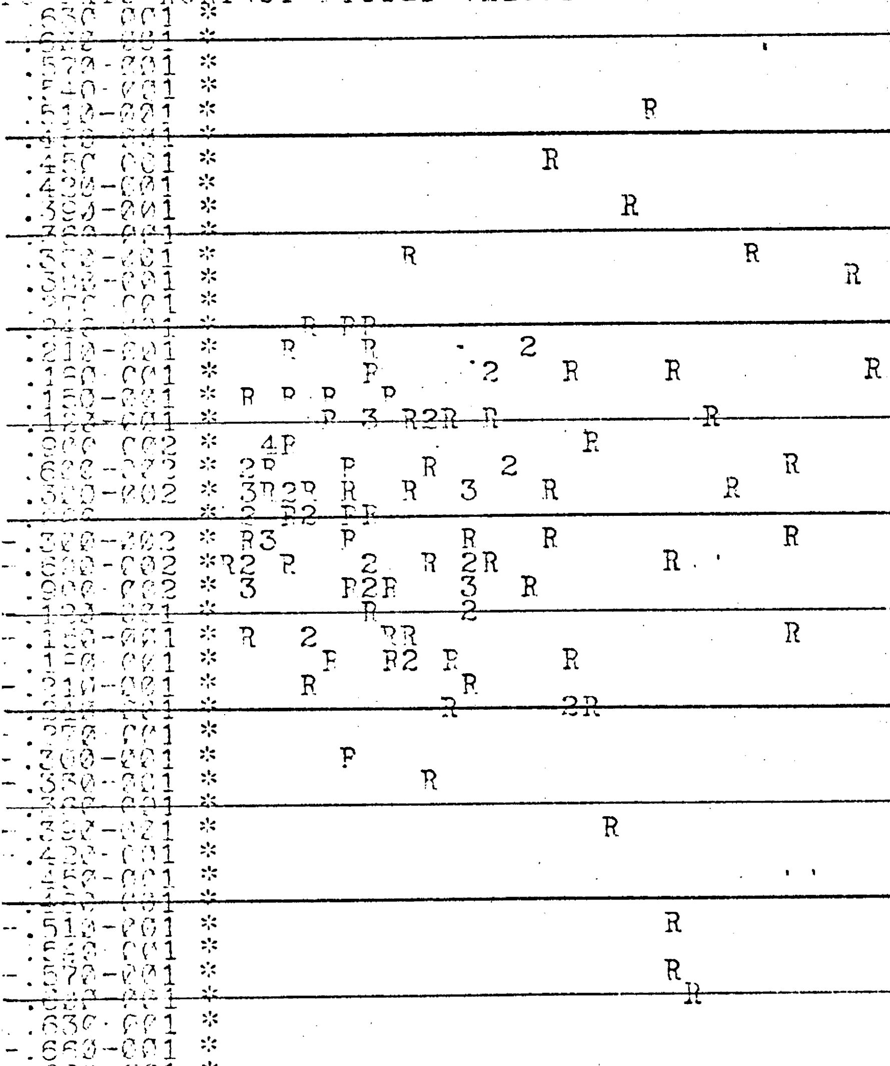
TSS 430.3 DF 124

REGRESSION	ESS	DF
1. Maximum model with different slopes and different intercepts.	76.23	113
2. Restricted model with different slopes and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	77.98	116
ii) 3 and 4 combined, 1 and 2 separate.	78.64	116
iii) 1 and 2 combined, 3 and 4 combined.	80.39	119
3. Maximum model with same slope and different intercepts.	87.26	119*
4. Restricted model with same slope and different intercepts.		
i) 1 and 2 combined, 3 and 4 separate.	87.92	120*
ii) 3 and 4 combined, 1 and 2 separate.	88.52	120*
iii) 1 and 2 combined, 3 and 4 combined.	89.19	121*
5. Maximum model with same intercept and different slopes.	77.40	116
6. Restricted model with same intercept and different slopes.		
i) 1 and 2 combined, 3 and 4 separate.	78.80	118
ii) 3 and 4 combined, 1 and 2 separate.	78.98	118
iii) 1 and 2 combined, 3 and 4 combined.	80.44	120

1, 2, 3, 4 indicate respective Species Group:

* Indicates regressions with all the coefficients significantly different from zero.

ESIDUALS AGAINST FITTED VALUES

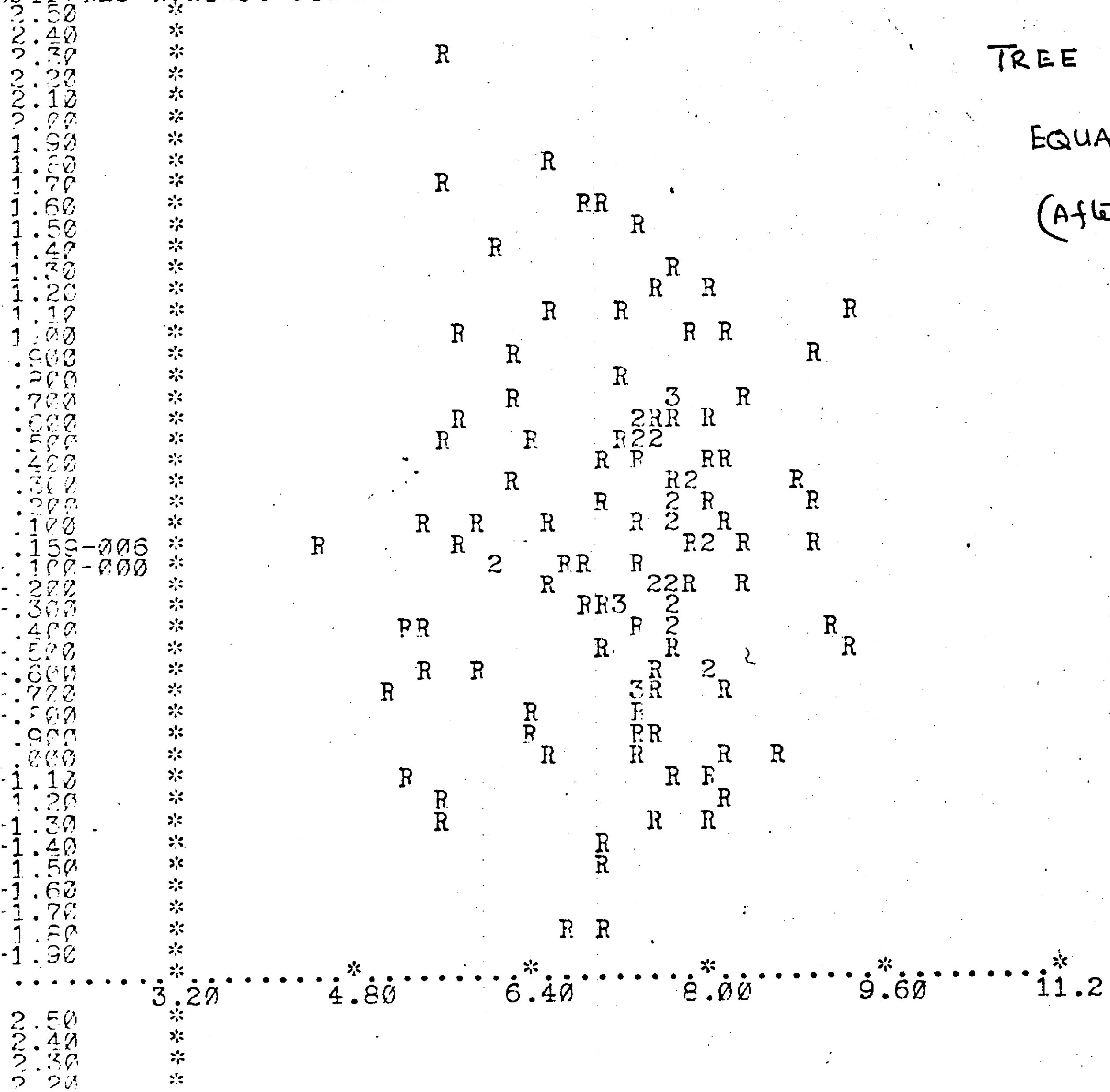


TREE HEIGHT AND DBH OVER BARK

EQUATION : $V = b_0 + b_1 H + b_2 D^2 + b_3 H^2 + b_4 D^2 H$

(Before Weighting)

SIDUALS AGAINST FITTED VALUES



TREE HEIGHT AND DBH OVER BARK

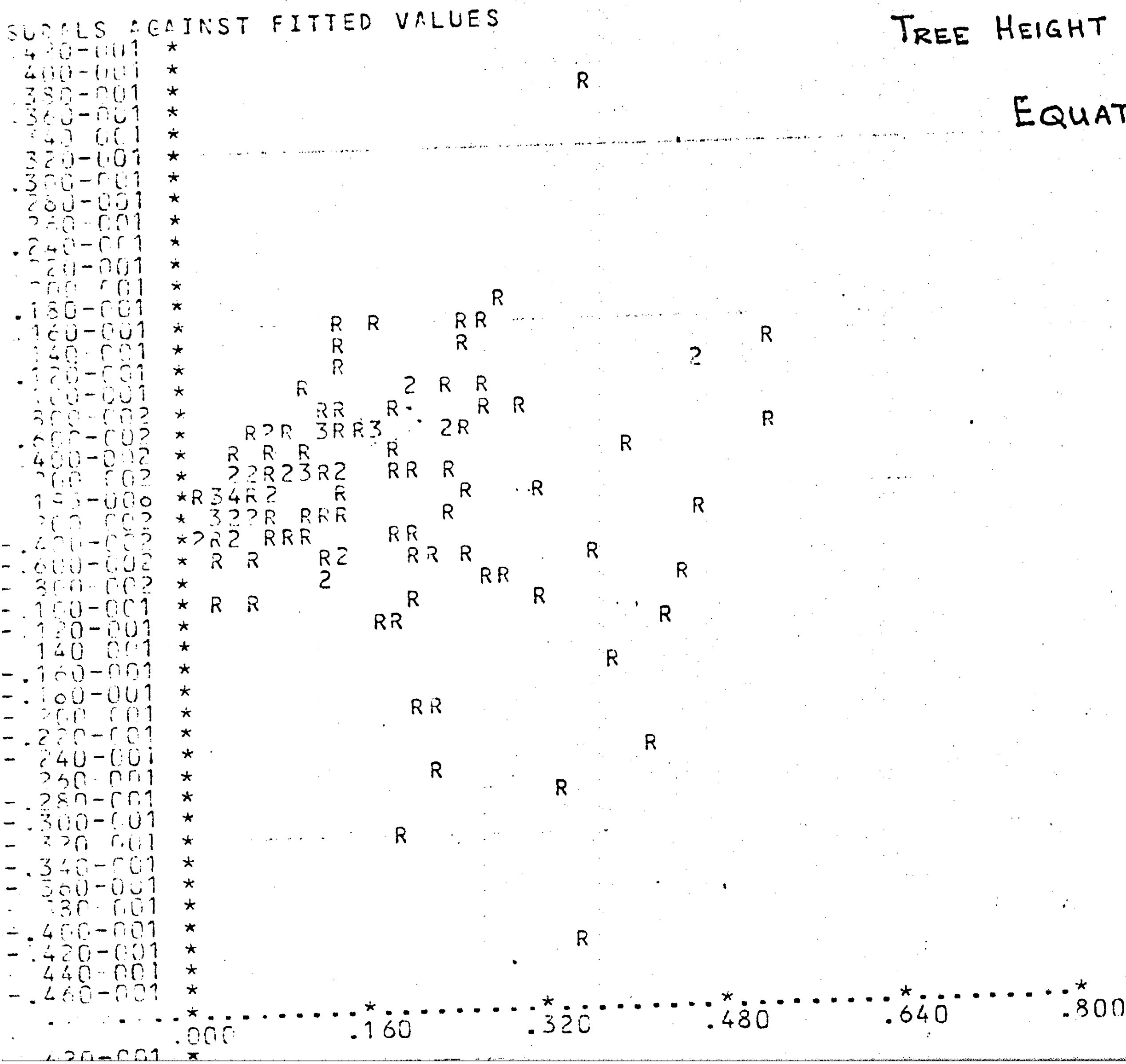
EQUATION : $V = b_0 + b_1 H + b_2 D^2 + b_3 H^2 + b_4 D^2 H$

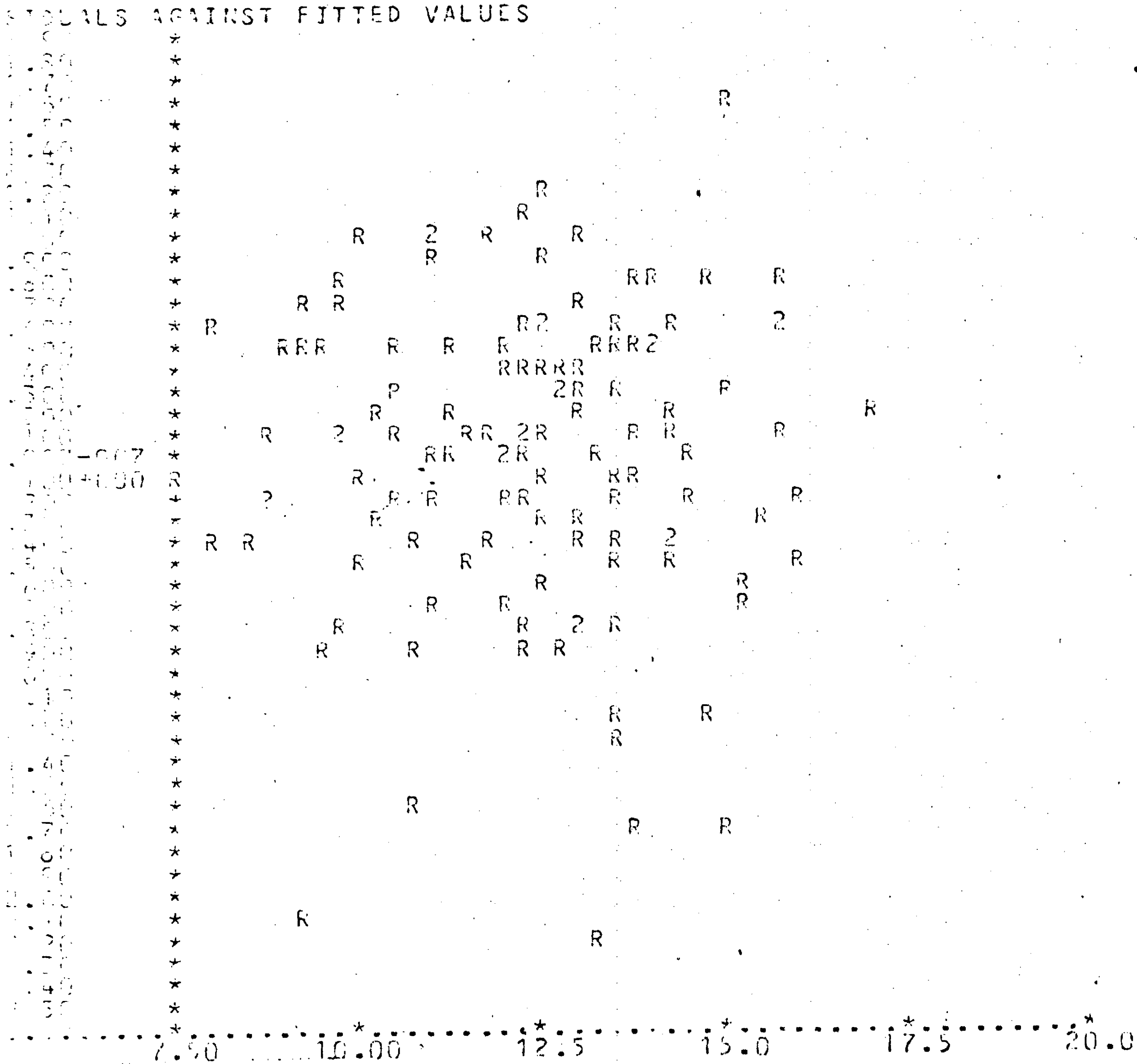
(After weighting)

TREE HEIGHT AND DBH UNDER BARK

EQUATION : $V = b_0 + b_1 D^2H + b_2 D^3H + b_3 (D^2H)^2$

(Before weighting)

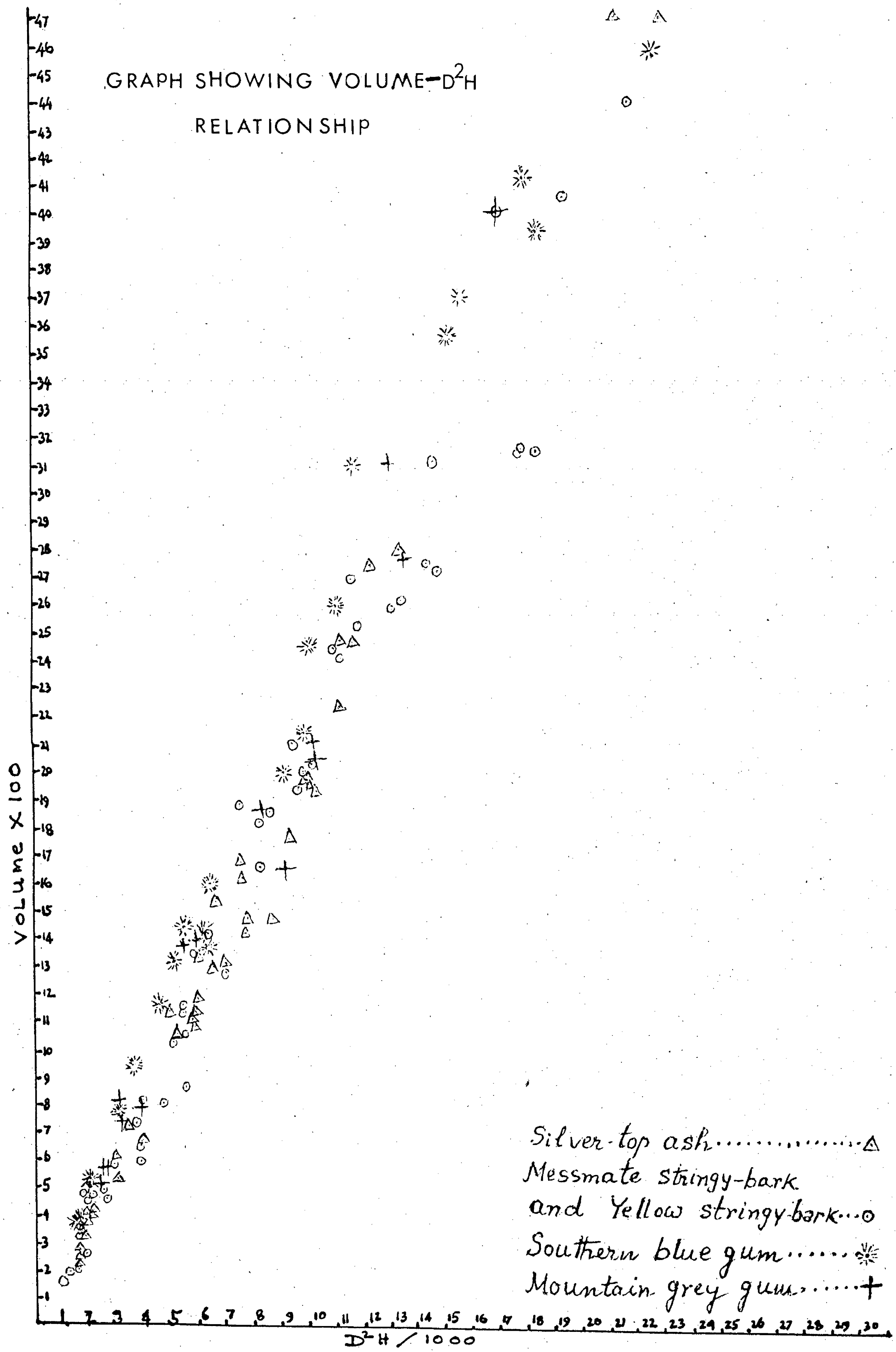




TREE HEIGHT AND DBH UNDER BARK

EQUATION : $V=b_0+b_1D^2H+b_2D^3H+b_3(D^2H)^2$

(After Weighting)


GRAPH SHOWING VOLUME-D^2H
RELATIONSHIP
VOLUME X 100
D^2H / 1000
Silver-top ash .... △
Messmate stringy-bark
and Yellow stringy-bark ... ⊙
Southern blue gum ...... ✳
Mountain grey gum ...... +